Lecture 1

Electrical Quantities

Quantity	Quantity Symbol	SI Unit	SI Unit Symbol
Current	I	Ampere	A
Charge	Q	Coulomb	C
Voltage	V	Volt	V
Resistance	R	Ohm	Ω
Conductance	G	Siemens	S
Real Power	P	Watt	W
Reactive Power	Q	Var	VAR
Apparent Power	S	Volt-ampere	VA
frequency	f	Hertz	Hz
inductance	L	Henry	H
reactance	X	Ohm	Ω
Capacitance	C	Farad	F
impedance	Z	Ohm	Ω
Length	l	Meter	m
Force	F	Newton	N
Energy	W	Joule	J=N.m
Temperature	T	Kelvin	K
Time	t	Second	s

Metric Prefixes

Metric Prefix	Symbol	Power of ten
Pico	p	10^{-12}
Nano	n	10^{-9}
Micro	µ	10^{-6}
Milli	m	10^{-3}
Centi	c	10^{-2}
Kilo	k	10^{3}
Mega	M	10^{6}
Giga	G	10^{9}
Tera	T	10^{12}

Electric Current

Electric Current (I): The amount of electric charge flowing through the cross-sectional surface of the conductor over time.

The SI unit of Electric current is Ampere (A).

If 6.242×10^{18} *electrons* drift at uniform velocity through the imaginary circular cross section in 1 *second* , the flow of charge, or current, is said to be 1 ampere (A).

One coulomb of electric charge is equal to 6.242×10^{18} *electrons* (i.e. 1 C= 6.242×10^{18} *electrons*).

Mathematically, the current in amperes can be calculated using the following equation:

$$\boldsymbol{I = \frac{Q}{t}}$$

Where Q is the electric charge in coulombs (C), that flows at time duration of *t*.

***t:* is the time duration in seconds (s).**

Example 1: In a copper wire a flow of charge is 0.12 C in a time of 58 ms . Find the current in this wire ?

Solution:

$$\boldsymbol{I = \frac{Q}{t} = \frac{0.12}{58 \times 10^{-3}} = 2.0689\ A}$$

Example 2: The charge flowing through the imaginary surface is 0.16 C every 64 ms . Determine the current in amperes ?

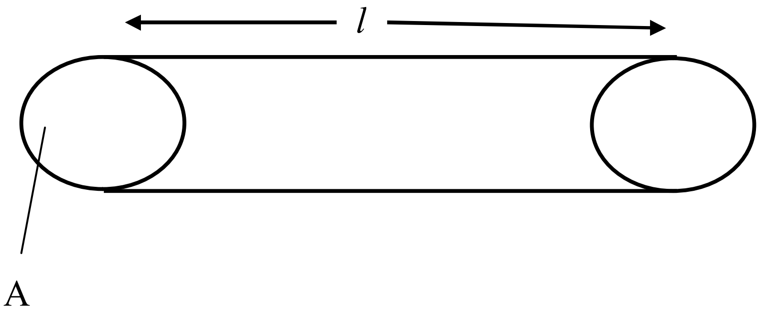
Example 3: Determine the time required for $\mathbf{4} \times \mathbf{10^{16}}$ electrons to pass through the imaginary surface if the current is 5 mA?

Resistance :-

The unit of resistance (R) is ohm (Ω)

The resistance of any material is depends on four factors:

1. The length of conductor (l) ⇒ $R \propto l$
2. Cross –section area of conductor (A) ⇒ $R \propto \frac{1}{A}$
3. The nature of material conductor.
4. The temperature of conductor.



$R \propto \frac{l}{A}$

$$R = \rho \frac{l}{A}$$

ρ= constant is called resistivity or specific resistance unit of resistivity.

$$R = \rho \frac{l}{A} \implies \rho = \frac{RA}{l} = \frac{\boldsymbol{\Omega . m^2}}{\boldsymbol{m}} = \Omega . m \text{ or } \Omega . cm$$

$$A = \pi r^2 = \pi \left(\frac{d}{2}\right)^2$$

where

r = radians of section.

d = diameter of section.

Conductance (G):-

$$G = \frac{1}{R} = \frac{1}{\frac{\rho l}{A}} = \frac{A}{\rho l} \text{ (Siemens (S))}$$

Example: What is the resistance of 3 Km length of wire with cross section area 6 mm^2 and resistivity 1.8 μΩcm .

Solution:

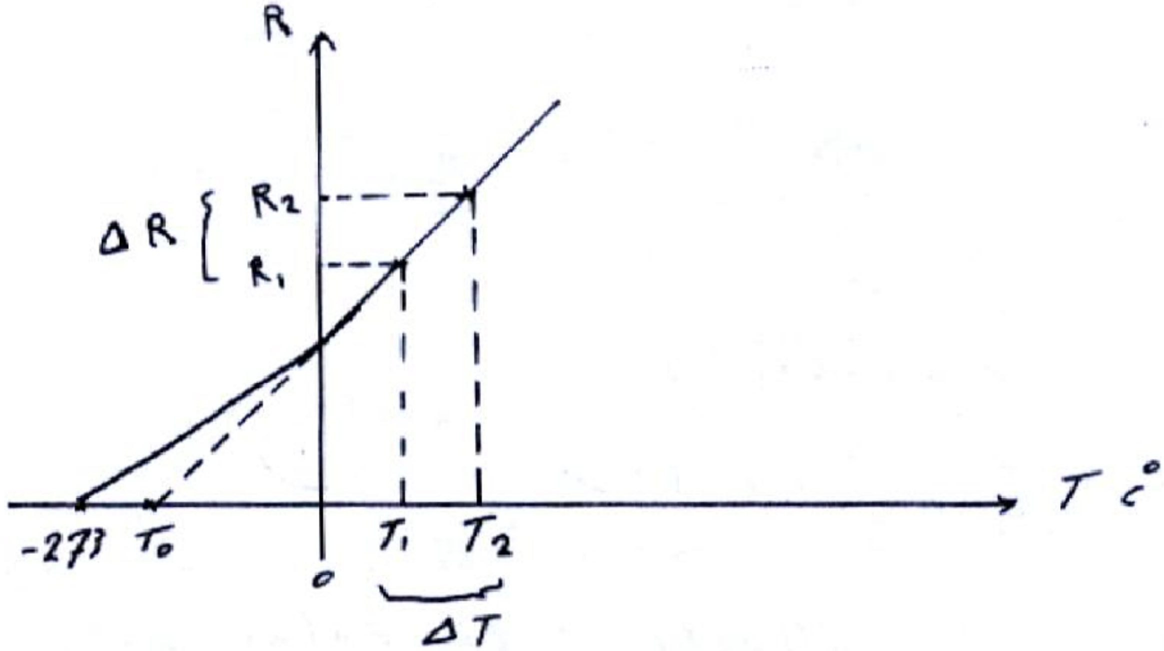
$$R = \frac{\rho l}{A} = \frac{(1.8 \times 10^{-6} \times 10^{-2}) \times (3 \times 10^{3})}{(6 \times 10^{-6})} = 9\ \Omega$$

Example: What is the resistance of 100 m length of copper wire with a diameter of (1 mm) and resistivity 0.0159 μΩm .

Solution:

$$R = \frac{\rho l}{A} = \frac{\rho l}{\pi \left(\frac{d}{2}\right)^2} = \frac{(0.0159 \times 10^{-6}) \times 100}{\pi \left(\frac{1 \times 10^{-3}}{2}\right)^2} = 2.02\ \Omega$$

Effect of temperature on a resistance



$$Slop = \frac{\Delta R}{\Delta T} = constant = \frac{R_2 - R_1}{T_2 - T_1} = \frac{R_2 - R}{T_2 - T} = \frac{R - R_1}{T - T_1}$$

Example: The resistance of material is 300 Ω at 10 C°, and 400 Ω at 60 C°. Find its resistance at 50 C° ?

Solution:

$$Slop = \frac{R_2 - R_1}{T_2 - T_1} = \frac{400 - 300}{60 - 10} = 2\ \Omega/C^0$$

$$2 = \frac{R - R_1}{T - T_1} = \frac{R - 300}{50 - 10} = \frac{R - 300}{40}$$

$$R - 300 = 80 \rightarrow R = 80 + 300 \ \rightarrow R = 380\ \Omega$$

Voltage (Potential Difference)

Electric voltage is a difference of potential between two places with different charges. Voltage provides the ability to move charges and hence do a work, and therefore voltage is also sometimes called electromotive force (EMF).

The symbol for voltage is V or sometimes U (v or u, if the voltage is time varying quantity).

The measurement unit of voltage is volt (V).

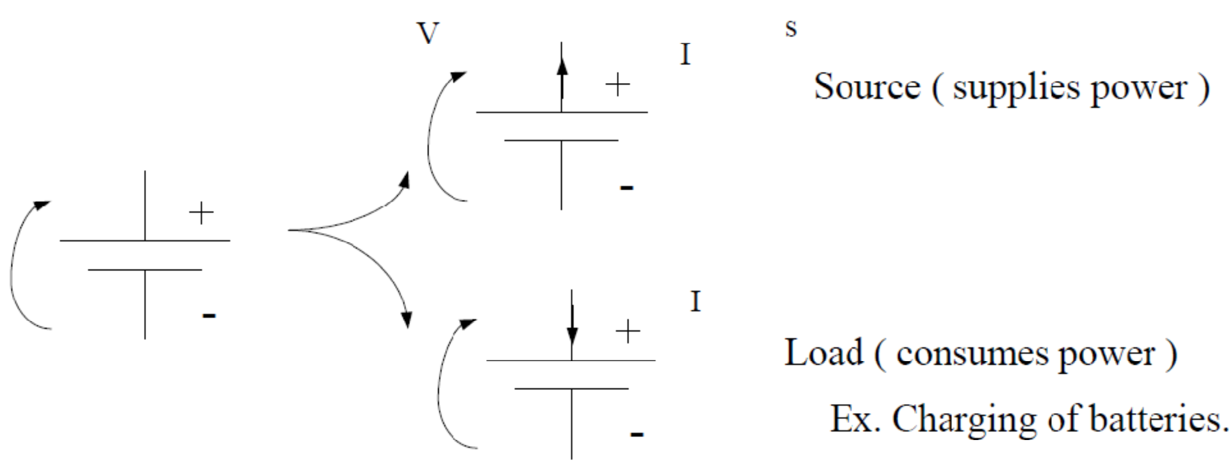
The SI definition for volt is " The volt is the potential difference between two points of a conducting wire carrying a constant current of 1 ampere, when the power dissipated between these points is equal to 1 watt".

Electrical Circuits :-

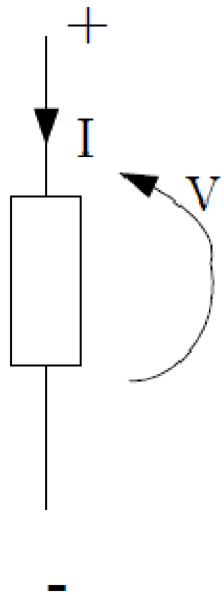
Circuit element :- is a two – terminal electrical component .
Electrical circuit :- Interconnected group of elements .

Sources of DC Voltage

Source of emf (electro motive force)



Source = current and voltage in the same direction.
load = current and voltage in the opposite direction .
Resistive element, is always load and always dissipates power.

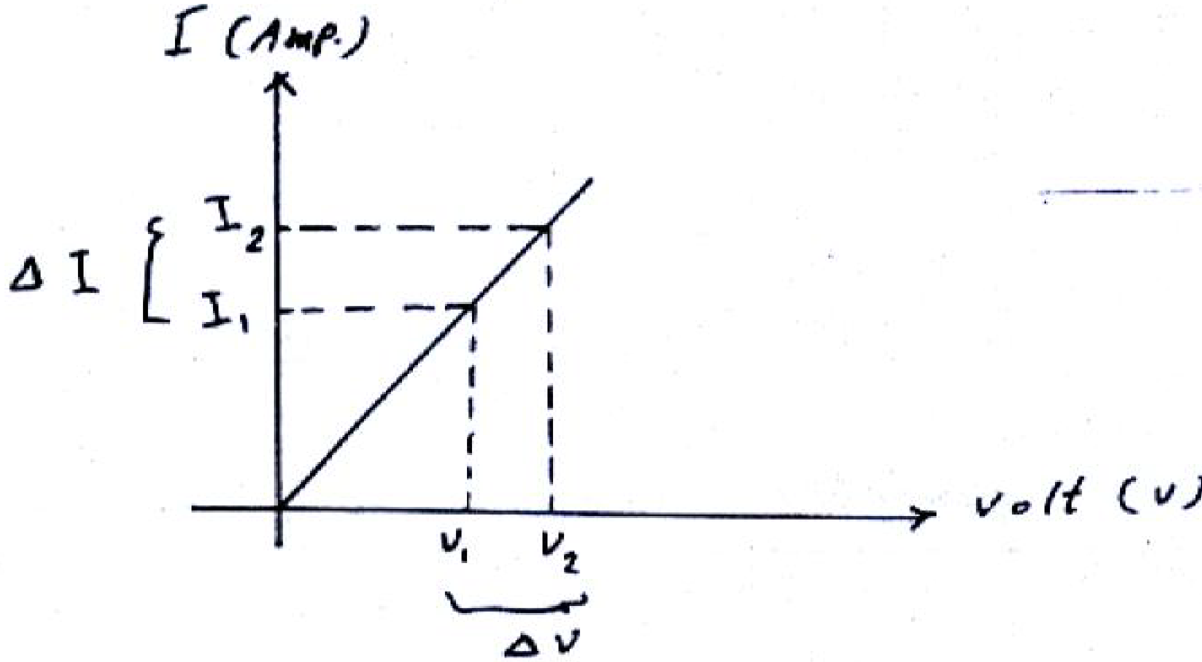


Load dissipates power (Ex. Resistive load)

P= VI in watts)

Lecture 2

Ohm's Law **:-** Ohm's law states that the voltages (V) across a resistor (R) is directly proportional to the current (I) flowing through the resistor .



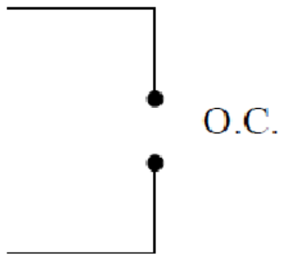
$$\text{Slop} = \frac{\Delta I}{\Delta V} = \frac{1}{R}$$

$$\frac{V}{I} = \text{constant} = \text{R}$$

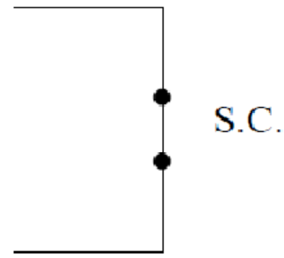
$$\boxed{R = \frac{V}{I}}$$

$$\Omega \;;\; \rightarrow \; \text{V} = \text{I . R} \;\;;\;\; I = \frac{V}{R}$$

❖ The resistance of short circuit element is approaching to zero (I=∞).
❖ The resistance of open circuit is approaching to infinity (I=0).



$$R = \infty \qquad G = \frac{1}{R} = 0$$



$$R = 0 \qquad G = \frac{1}{R} = \infty$$

Hence $G = \frac{1}{R} = \frac{I}{V}$ Siemens (S) or mhos (υ) .

<u>Electrical Energy (*W*) and Power (*P*) :-</u>

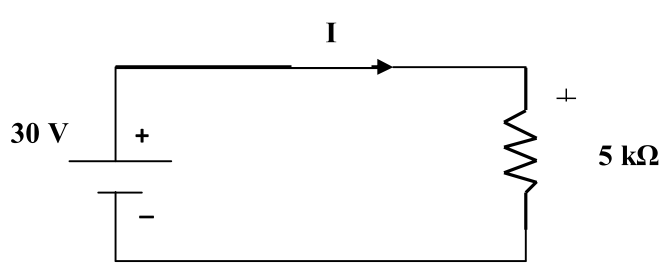
$$P = \frac{W}{t} \rightarrow W = P \times t \quad \text{kWh}$$

$$P = (V \times I) = (I^2 \times R) = \left(\frac{V^2}{R}\right) \quad \text{W}$$

$$W = P \times t = (V \times I) \times t = (I^2 \times R) \times t = \left(\frac{V^2}{R}\right) \times t$$

$$Energy\ in\ \text{kWh} = \text{Power}\ (P)\ \times \text{time}\ (t)/1000$$

Example : For the following circuit diagram , calculate the conductance and the power ?



Solution :

$$I = \frac{V}{R} = \frac{30}{5\times10^3} = 6 \text{ m}A$$

$$G = \frac{1}{R} = \frac{1}{5\times10^3} = 0.2 \text{ ms}$$

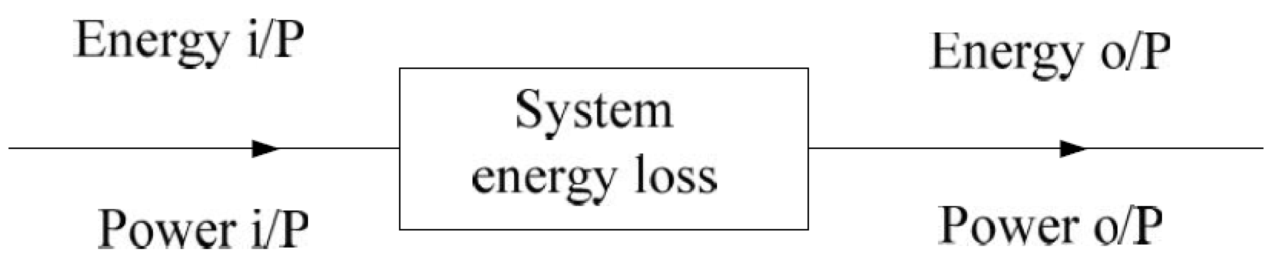
$$P = V.I = (6 \times 10^{-3}) \times 30 = 180\ m\text{W}$$

$$or\ P = I^2.R = (6 \times 10^{-3})^2 \times 5 \times 10^3 = 180\ m\text{W}$$

$$or\ P = V^2.G = (30)^2 \times 0.2 \times 10^{-3} = 180\ m\text{W}$$

$$or\ P = \frac{V^2}{R} = \frac{(30)^2}{5 \times 10^3} = 180\ m\text{W}$$

<u>Efficiency (η) :-</u>



$$\text{W}_{\text{i/p}} = \text{W}_{\text{o/p}} + \text{W}_{\text{loss}}$$

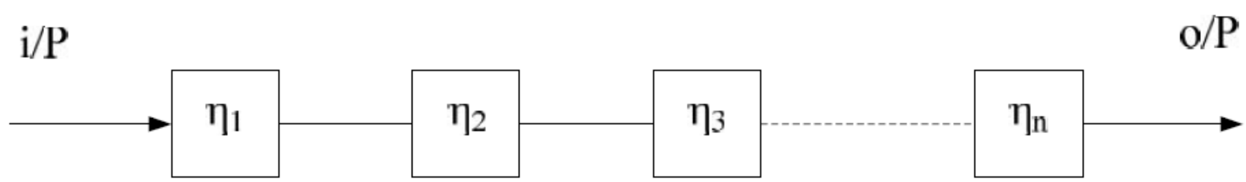
$$\frac{W_{i/p}}{t} = \frac{W_{o/p}}{t} + \frac{W_{loss}}{t}$$

$$\text{P}_{\text{i/p}} = \text{P}_{\text{o/p}} + \text{P}_{\text{loss}}$$

$$\text{Efficiency} (\eta) = \frac{\text{Output power}}{\text{Input power}} \times 100\%$$

$$\eta = \frac{P_o}{P_i} \times 100\%$$

$$\eta = \frac{W_o}{W_i} \times 100\%$$



$$\eta_T = \eta_1 \times \eta_2 \times \eta_3 \times \ldots\ldots \times \eta_n$$

Example: A 2 hp motor (output power) operates at an efficiency of 75 %, what is the power input in Watt, if the input current is (9.05) A, calculate also the input voltage?

Solution:

1 hours power (hp) = 746 Watt

$$\eta = \frac{P_o}{P_i} \times 100\%$$

$$0.75 = \frac{2 \times 746}{P_i} \Rightarrow P_i = \frac{1492}{0.75} = 1989.33W$$

$$P = E.I \Rightarrow E = \frac{P}{I} = \frac{1989.33}{9.05} = 219.82 \cong 220V$$

Example: What is the energy in kWh of using the following loads:-
a) 1200 W toaster for 30 min.
b) Six 50 W bulbs for 4 h.
c) 400 W washing machines for 45 min.
d) 4800 W electric clothes dryer for 20 min.

Solution :

$$W = \frac{P(W) \times t(h)}{1000}$$

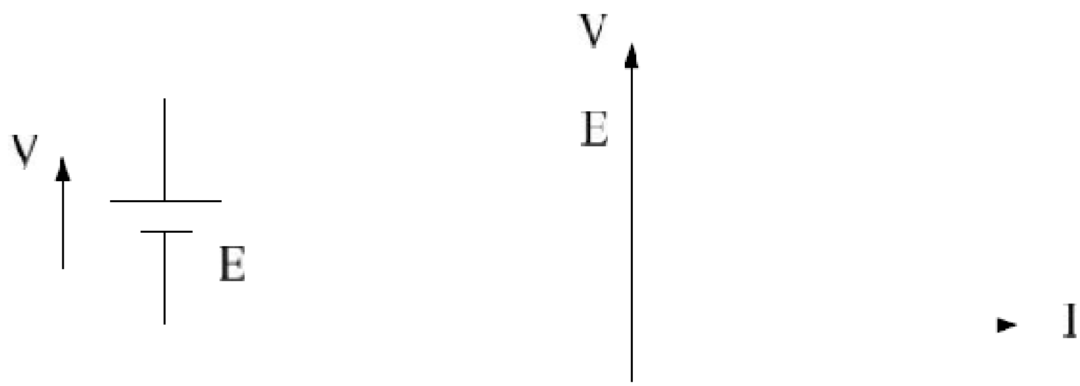
$$W = \frac{1200 \times \left(\frac{30}{60}\right) + 6 \times 50 \times 4 + 400 \times \left(\frac{45}{60}\right) + 4800 \times \left(\frac{20}{60}\right)}{1000}$$

$$= \frac{600 + 1200 + 300 + 1600}{1000} = \frac{3700}{1000} = 3.7 KWh$$

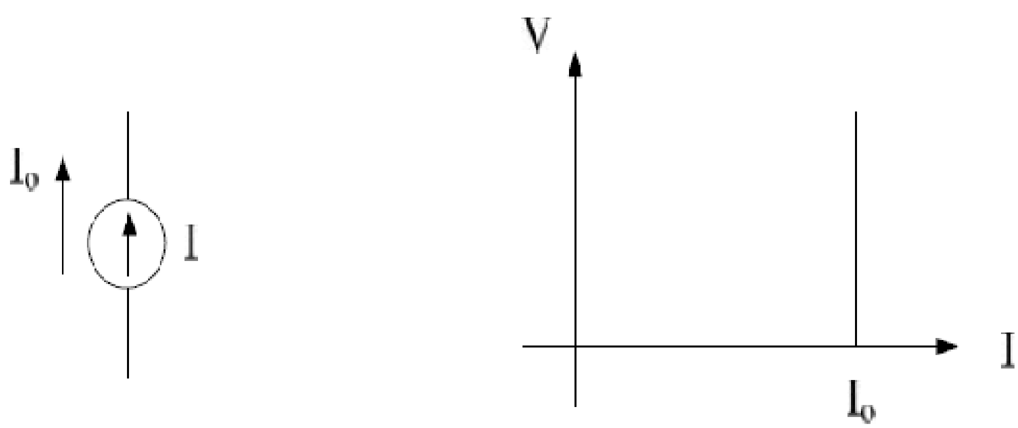
D.C. Sources:-

The d.c. sources can be classified to:-

1- Batteries : Voltage; Ampere – hours
2- Generators
3- Photo cells.
4- Rectifiers .



$V = E = \text{constant voltage element}$



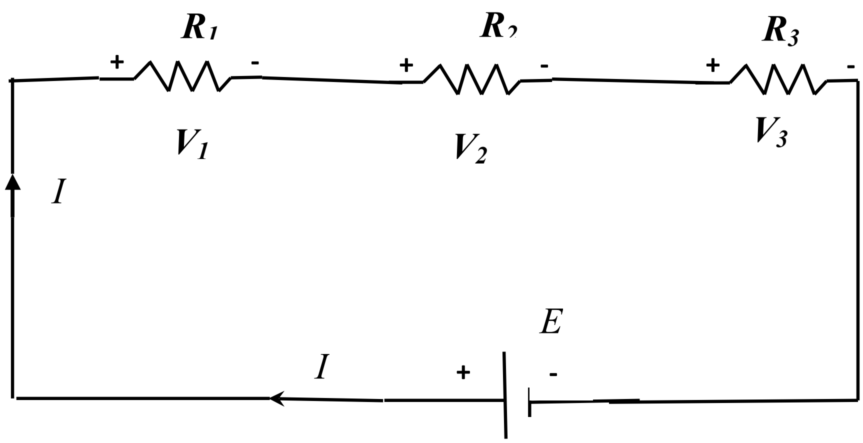
$I = I_o = \text{constant current element}$

- Voltage Source generates voltage and current (voltage is constant).
- Current Source generates current and voltage (current is constant).

Lecture 3

Series Circuits :-

Resistors in Series



$V_1 = I.R_1$

$V_2 = I.R_2$

$V_3 = I.R_3$

$-E + V_1 + V_2 + V_3 = 0 \ \rightarrow \ E = V_1 + V_2 + V_3$

$\therefore \ E = I.R_1 + I.R_2 + I.R_3$

$\therefore \ E = I.[R_1 + R_2 + R_3] = I.R_T$

The current in the series circuit is the same through each series element &

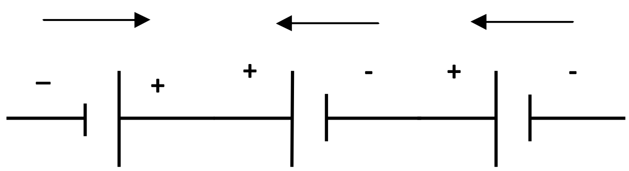
$\therefore \ R_T = R_1 + R_2 + R_3 + \cdots \cdots + R_N$

$$I = \frac{E}{R_T} = \frac{V_1}{R_1} = \frac{V_2}{R_2} = \frac{V_3}{R_3}$$

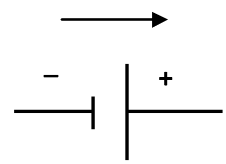
$$P_t = P_1 + P_2 + P_3 = E.I$$

$$E.I = V_1.I + V_2.I + V_3.I$$

Voltage Sources in Series



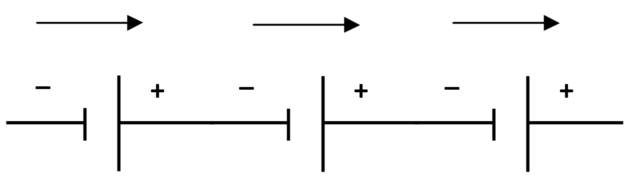
E_1	E_2	E_3
20 V	10 V	3 V



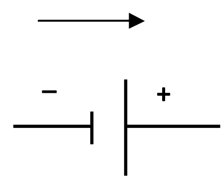
E_T

$E_T = E_1 - E_2 - E_3$

$E_T = 20 - 10 - 3 = 7$ V



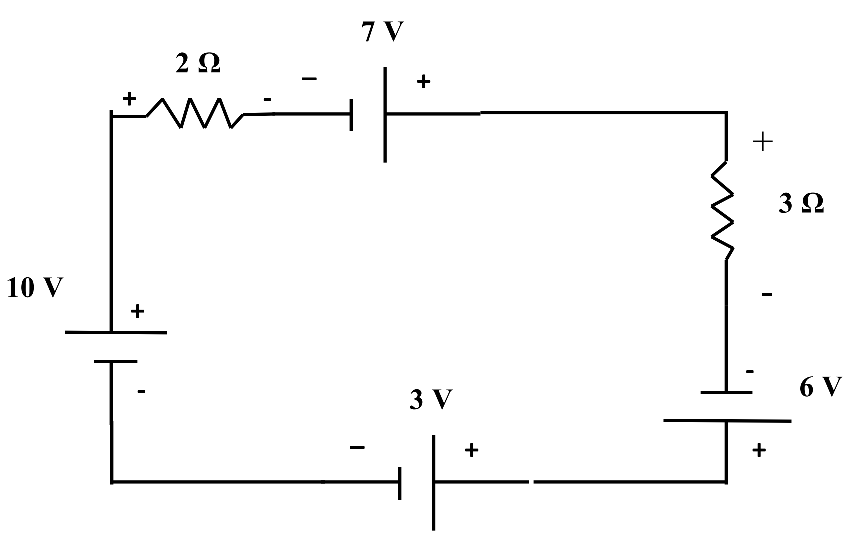
E_1	E_2	E_3
5 V	6 V	7 V



$E_T = E_1 + E_2 + E_3$

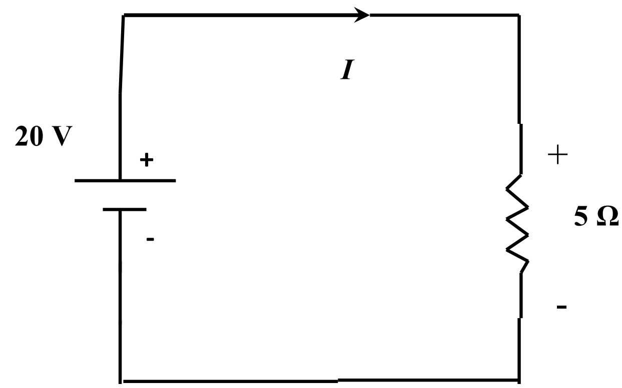
$E_T = 5+6+7 = 18$ V

Example: Find the current for the following circuit diagram?



$E_T = 10 + 7 + 6 - 3 = 20\ \text{V}$

$R_T = 2 + 3 = 5\ \Omega$



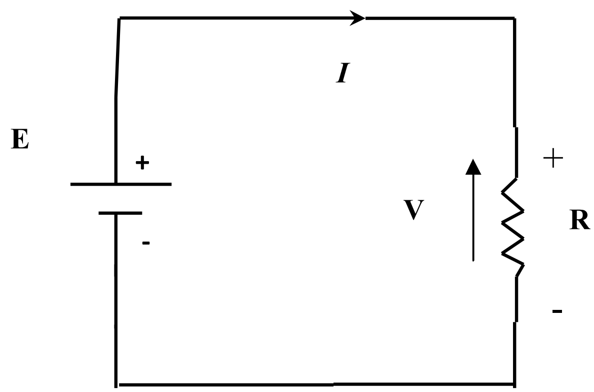
$$I = I_T = \frac{E_T}{R_T} = \frac{20}{5} = 4\ \text{A}$$

<u>Kirchoff's voltage law (K.V.L.):-</u>

The algebraic sum of all voltages around any closed path is zero.

$$\sum_{i=1}^{N} V_i = 0$$

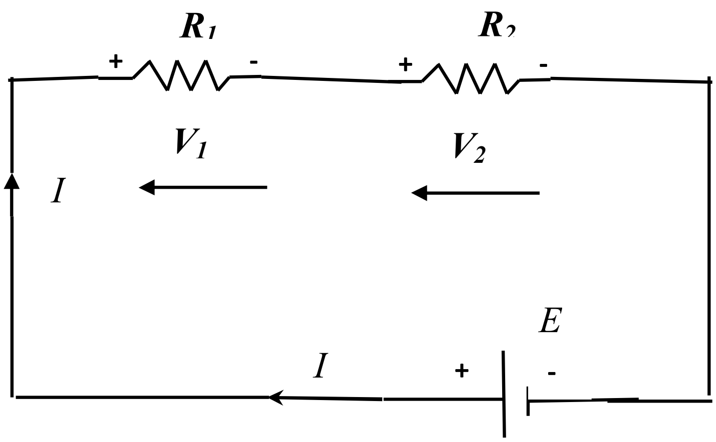
Where N is the number of voltages in the path (loop) , and V_i is the i^{th} voltage .



$$-E + V = 0$$

$$E = V = I.R$$

$$I = \frac{E}{R} = \frac{V}{R}$$



$$-E + V_1 + V_2 = 0$$

$$E = V_1 + V_2$$

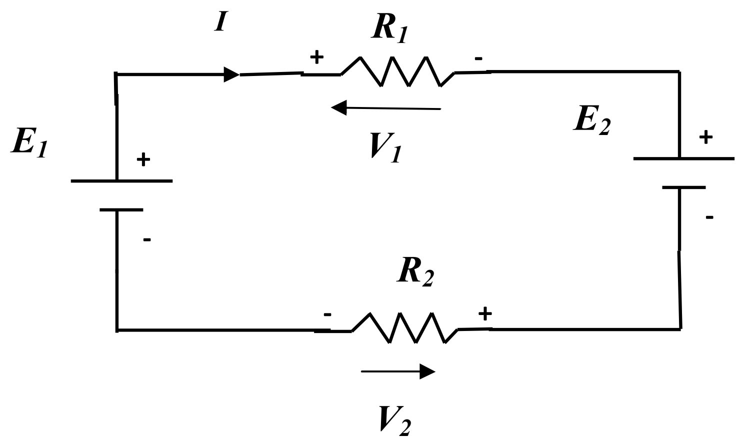
$$E = V_1 + V_2 = I.R_1 + I.R_2 = I.(R_1 + R_2) = I.R_T$$

$$R_T = R_1 + R_2$$

$$I = \frac{E}{R_T} = \frac{V_1 + V_2}{R_T}$$

Lecture 4

Example: Use K.V.L. to find the current in the following circuit diagram?



Solution:

From K.V.L. $\sum V = 0$

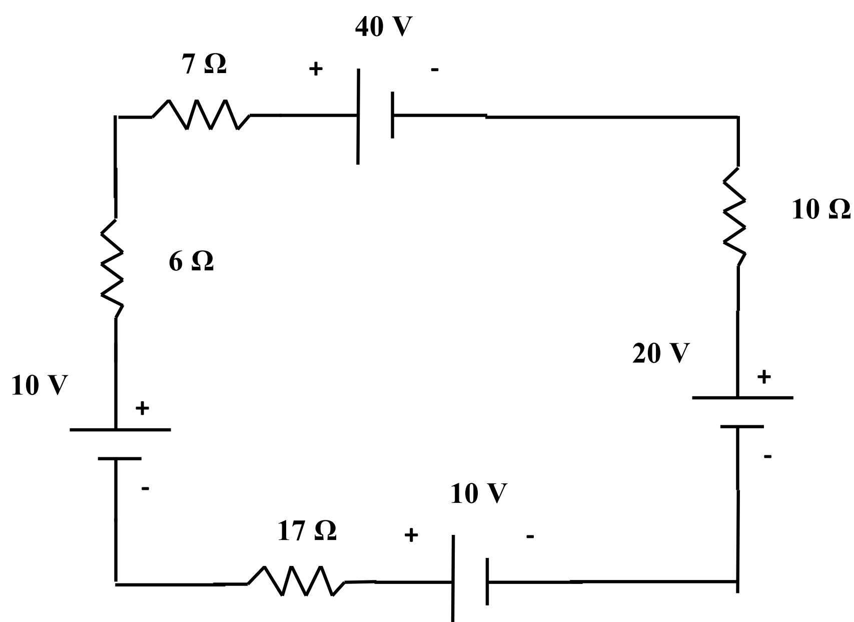
$$-E_1 + V_1 + E_2 + V_2 = 0$$

$$E_1 - E_2 = V_1 + V_2 = I.R_1 + I.R_2 = I.(R_1 + R_2)$$

$$\therefore I = \frac{E_1 - E_2}{R_1 + R_2} = \frac{E_1 - E_2}{R_T}$$

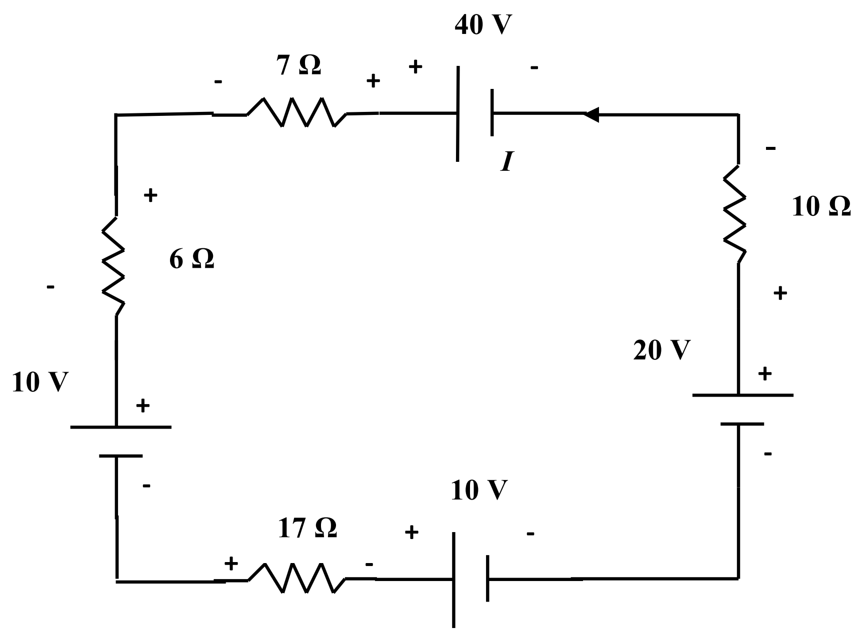
Example: For the following circuit diagram, Find *I* using:-

a) Ohm's law.
b) K.V.L.



Solution:

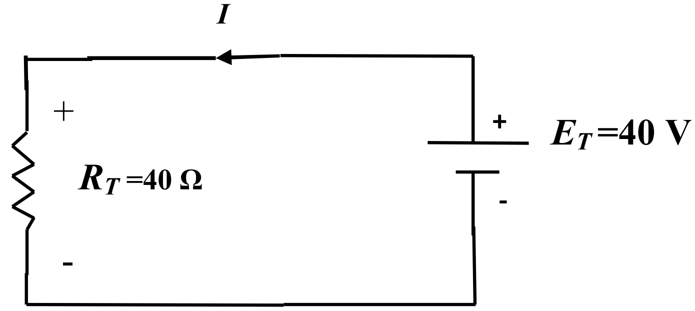
We need to identify the direction of the current



a) By applying ohm's law :-

$$I = \frac{E_T}{R_T} = \frac{20 + 40 - 10 - 10}{10 + 7 + 6 + 17} = \frac{40}{40} = 1\ \text{A}$$

The equivalent circuit



b) By applying K.V.L. :-

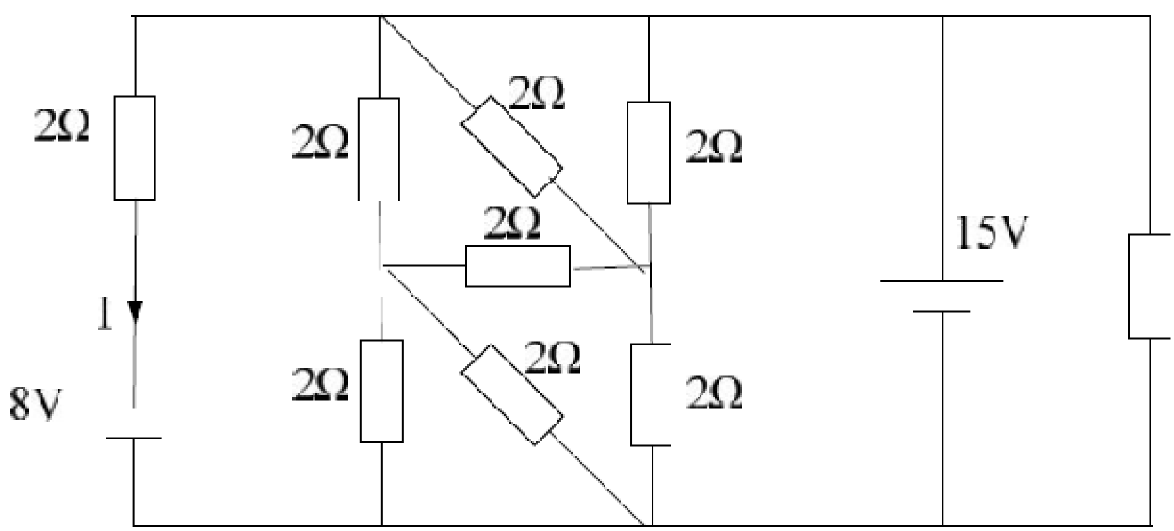
$$-10-6I-7I+40-10I+20-10-17I=0$$

$$-10+40+20-10-6I-7I-10I-17I=0$$

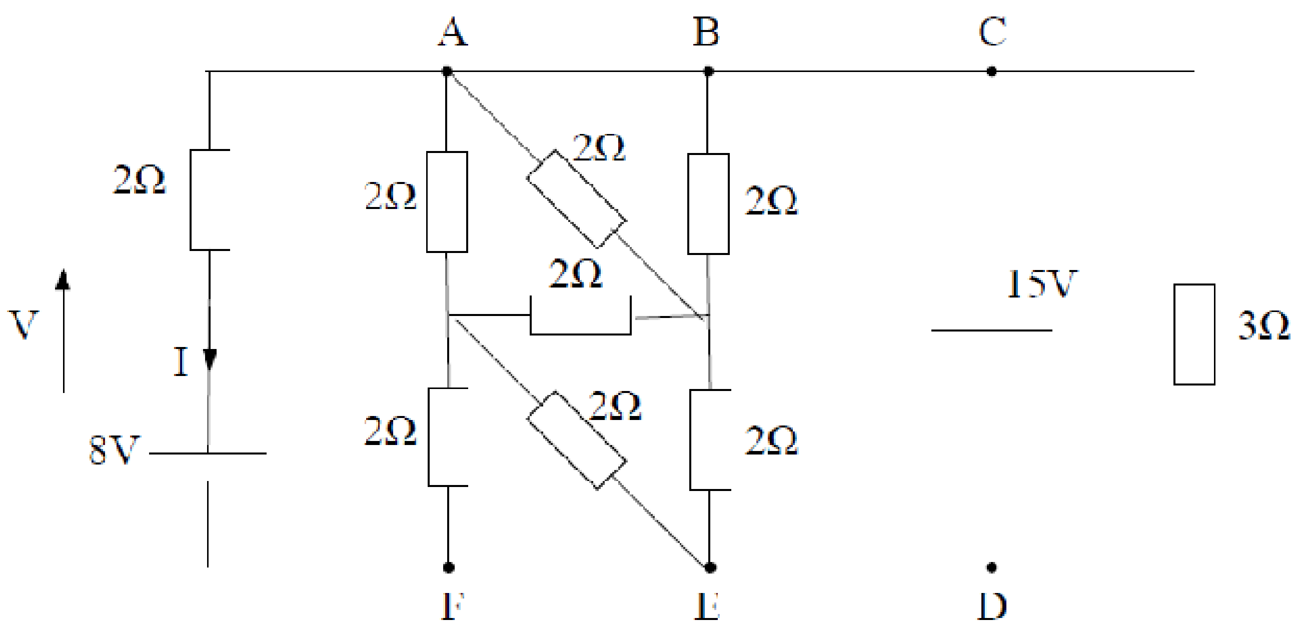
$$40-I(6+7+10+17)=0$$

$$40=40I \quad \rightarrow I=\frac{40}{40}=1\text{ A}$$

Example :- For the following circuit diagram , find the current ?



Solution:



Take the loop FABCDEF

$$-8 - 2I + 15 = 0 \;\rightarrow 2I = 7 \rightarrow I = \frac{7}{2} = 3.5\ A$$

Or

$$-8 - V_{2\Omega} + 15 = 0 \;\rightarrow V_{2\Omega} = 7\ V$$

$$V_{2\Omega} = IR$$

$$7 = I.2 \rightarrow I = \frac{7}{2} = 3.5\ A$$

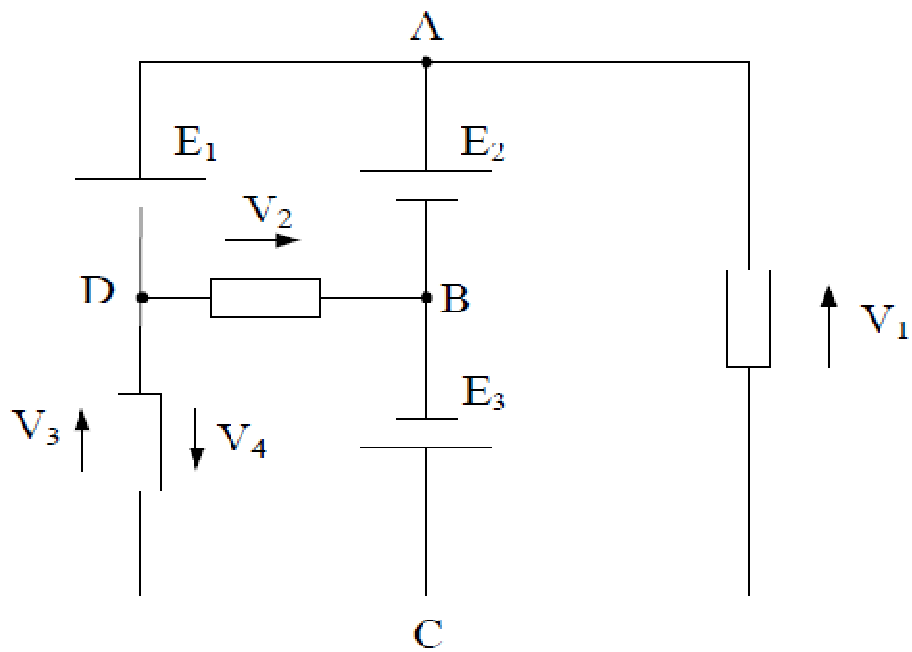
Definitions :-

Node :- Meeting point of 3 or more branches .

Branch :- Series of elements carrying the same current .

Loop :- Is any closed path in a circuit .

Hence for the loop circuit, we can find :-



4 nodes and 6 branches

Take the loop CBAC ; to find V_1

$E_3 - E_2 + V_1 = 0 \rightarrow V_1 = E_2 - E_3$

Or take the loop CABC

$-V_1 + E_2 - E_3 = 0 \rightarrow V_1 = E_2 - E_3$

Take the loop DABD ; to find V_2

$-E_1 + E_2 + V_2 = 0 \rightarrow V_2 = E_1 - E_2$

Take the loop CDABC ; to find V_3

$-V_3 - E_1 + E_2 - E_3 = 0 \rightarrow V_3 = E_2 - E_3 - E_1$

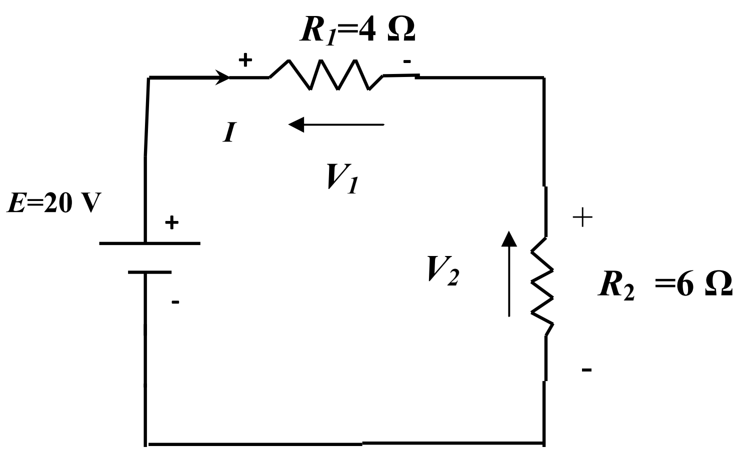
$V_4 = -V_3$

or

$$V_4 - E_1 + E_2 - E_3 = 0 \rightarrow V_4 = E_1 + E_3 - E_2$$

Lecture 5

Example :- For the following circuit diagram , find ; R_T , I , V_1 , V_2 , $P_{4\Omega}$, $P_{6\Omega}$, P_E , verify by K.V.L. ?



$$R_T = R_1 + R_2 = 4 + 6 = 10\ \Omega$$

$$I = \frac{E}{R_T} = \frac{20}{10} = 2A$$

$$V_1 = IR_1 = 2 \times 4 = 8V$$

$$V_2 = IR_2 = 2 \times 6 = 12V$$

$$P_{4\Omega} = I^2 R_1 = (2)^2 \times 4 = 16W \quad ; \text{ or } \quad P_{4\Omega} = \frac{V_1^2}{R_1} = \frac{(8)^2}{4} = 16w$$

$$P_{6\Omega} = I^2 R_2 = (2)^2 \times 6 = 24W \quad ; \text{ or } \quad P_{6\Omega} = \frac{V_2^2}{R_2} = \frac{(12)^2}{6} = 24w$$

$$P_E = IE = 2 \times 20 = 40W \quad ; \text{ or } \quad P_E = P_{4\Omega} + P_{6\Omega} = 16 + 24 = 40W$$

To verify results by using K.V.L. ; then

$$\sum_{i=1}^{N} V_i = 0$$

$$-E + V_1 + V_2 = 0$$

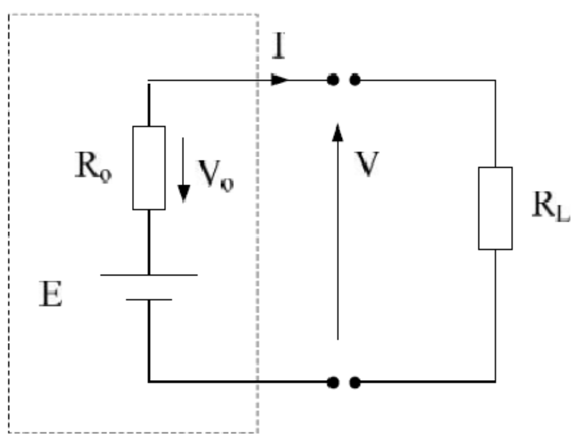
$$E = V_1 + V_2$$

$$20 = 8 + 12$$

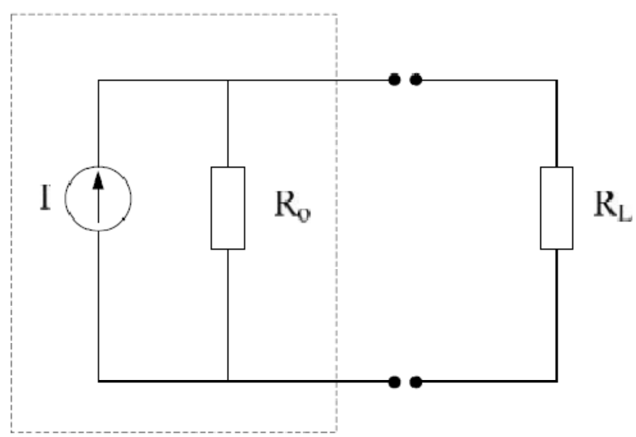
$20 = 20$ checks

Internal Resistance :-

Every practical voltage or current source has an internal resistance that adversely affects the operation of the source.
In a practical voltage source the internal resistance represent as a resistor in series with an ideal voltage source.
In a practical current source the internal resistance represent as a resistor in parallel with an ideal current source, as shown in the following figures.



Practical voltage source



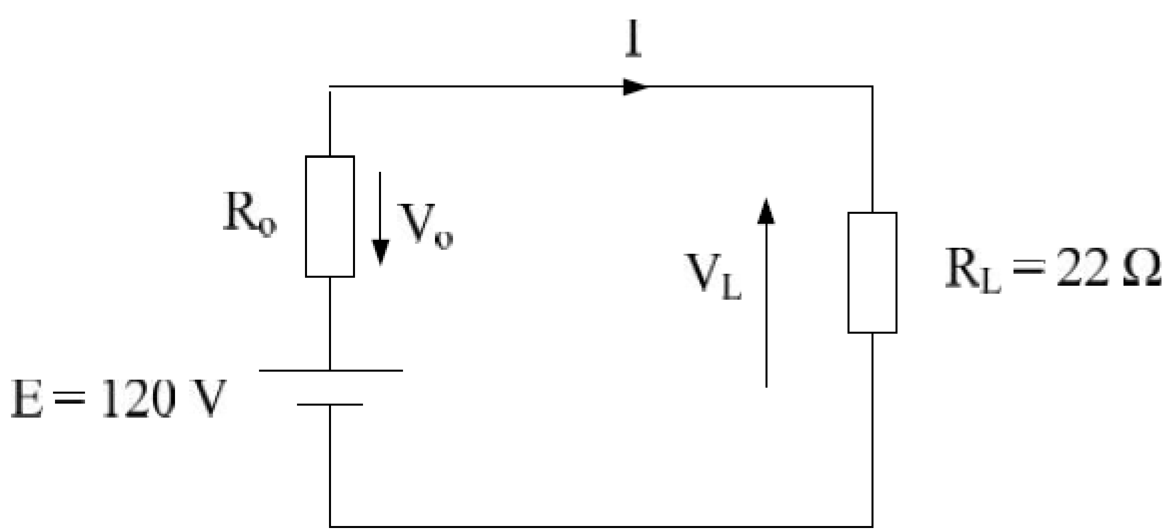
Practical current source

Where
R_o = Internal resistance
R_L = load resistance
According to K.V.L.

$$-E + V_o + V = 0$$
$$-E + IR_o + V = 0$$
$$V = E - IR_o$$

Note that an ideal sources have $R_o = 0$

Example :- For the following circuit diagram , calculate I and V_L for the following cases :-



a) $R_o = 0\ \Omega$
b) $R_o = 8\ \Omega$
c) $R_o = 16\ \Omega$

Solution :-

a.) By apply K.V.L.

$-E + V_o + V_L = 0$
$-120 + IR_o + IR_L = 0 \rightarrow -120 + 0 + 22I = 0$
$22I = 120 \rightarrow I = 5.4545$ A
$V_L = I.R_L$=$5.4545 \times 22 = 120$ V

b.) By apply K.V.L.

$-E + V_o + V_L = 0$
$-120 + IR_o + IR_L = 0 \rightarrow -120 + 8I + 22I = 0$
$30I = 120 \rightarrow I = 4$ A
$V_L = I.R_L$=$4 \times 22 = 88$ V

c.) By apply K.V.L.

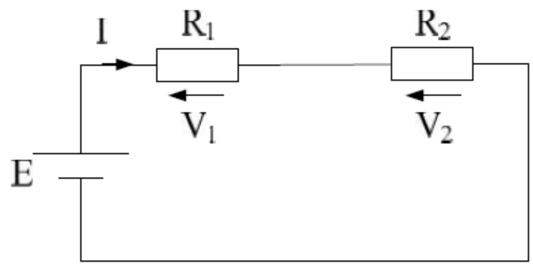
$-E + V_o + V_L = 0$
$-120 + IR_o + IR_L = 0 \rightarrow -120 + 16I + 22I = 0$
$38I = 120 \rightarrow I = 3.157$ A
$V_L = I.R_L$=$3.157 \times 22 = 69.47$ V

Then we can conclude that as R_o increase the total current and load voltage will decrease.

Lecture 6

Voltage divider Rule :-

$$R_T = R_1 + R_2$$



$$I = \frac{E}{R_T}$$

$$V_1 = I.R_1 = \left(\frac{E}{R_T}\right).R_1 = \frac{E.R_1}{R_T}$$

$$V_2 = I.R_2 = \left(\frac{E}{R_2}\right).R_2 = \frac{E.R_2}{R_T}$$

$$\boxed{V_n = \frac{ER_n}{R_T}}$$

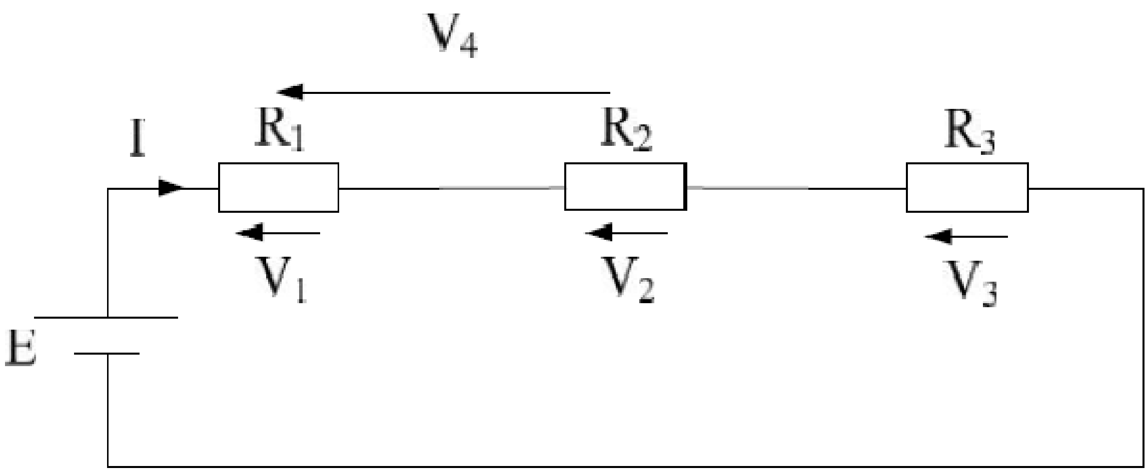
Voltage divider rule

V_n = Voltage across R_n

E = The (emf) voltage across the series elements .

R_T = The total resistance of the series circuits .

Example :- Using voltage divider rule , determine the voltage V_1 , V_2 , V_3 and V_4 for the series circuit in figure below , given that ; $R_1 = 2$ KΩ , $R_2 = 5$ KΩ , $R_3 = 8$ KΩ , E = 45 V ?



$$R_T = R_1 + R_2 + R_3 = 2 + 5 + 8 = 15 \text{ K}\Omega$$

$$V_1 = \frac{R_1 E}{R_T} = \frac{2 * 10^3 * 45}{15 * 10^3} = 6V$$

$$V_2 = \frac{R_2 E}{R_T} = \frac{5 * 10^3 * 45}{15 * 10^3} = 15V$$

$$V_3 = \frac{R_3 E}{R_T} = \frac{8 * 10^3 * 45}{15 * 10^3} = 24V$$

$$V_4 = \frac{(R_1 + R_2)E}{R_T} = \frac{7 * 10^3 * 45}{15 * 10^3} = 21V$$ or $V_4 = V_1 + V_2 = 21V$

To check $\quad -E + V_1 + V_2 + V_3 = 0$

$E = V_1 + V_2 + V_3 \rightarrow 45 = 6 + 15 + 24 \rightarrow 45 = 45$

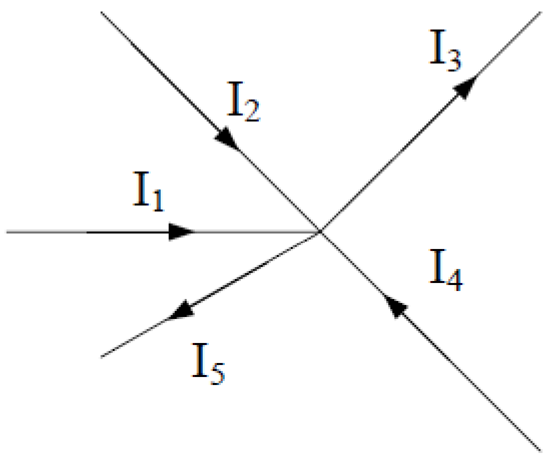
Kirchoff's Current Law (K.C.L.) :-

The algebraic sum of ingoing currents is equal to the outgoing currents at any point .

$$\sum I_{in} = \sum I_{out}$$

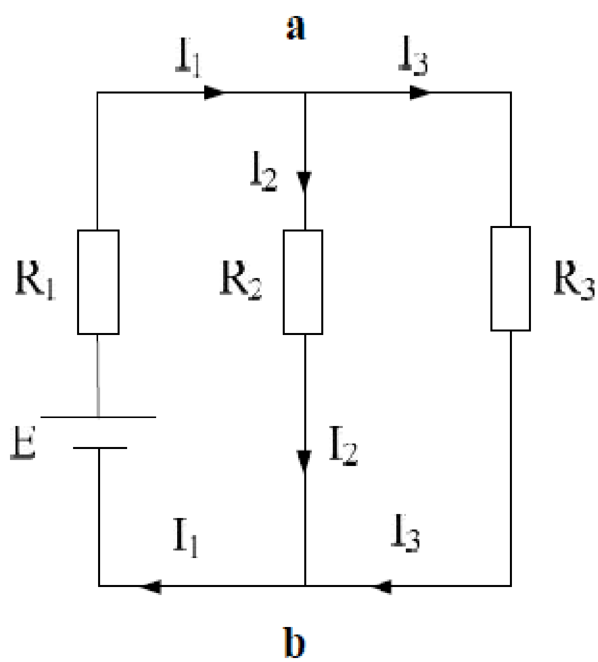
Or , At any point , the algebraic sum of entering and leaving current is zero .

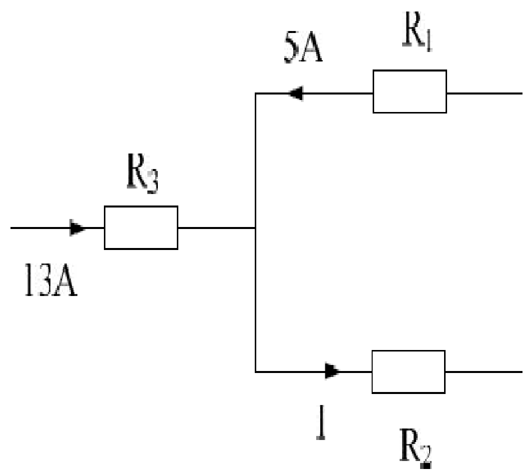
$$\sum I = 0$$



$$I_1 + I_2 + I_4 = I_3 + I_5$$

Or $\quad I_1 + I_2 + I_4 - I_3 - I_5 = 0$





At a

$$I_1 = I_2 + I_3$$

Or $\quad I_1 - I_2 - I_3 = 0$

At b

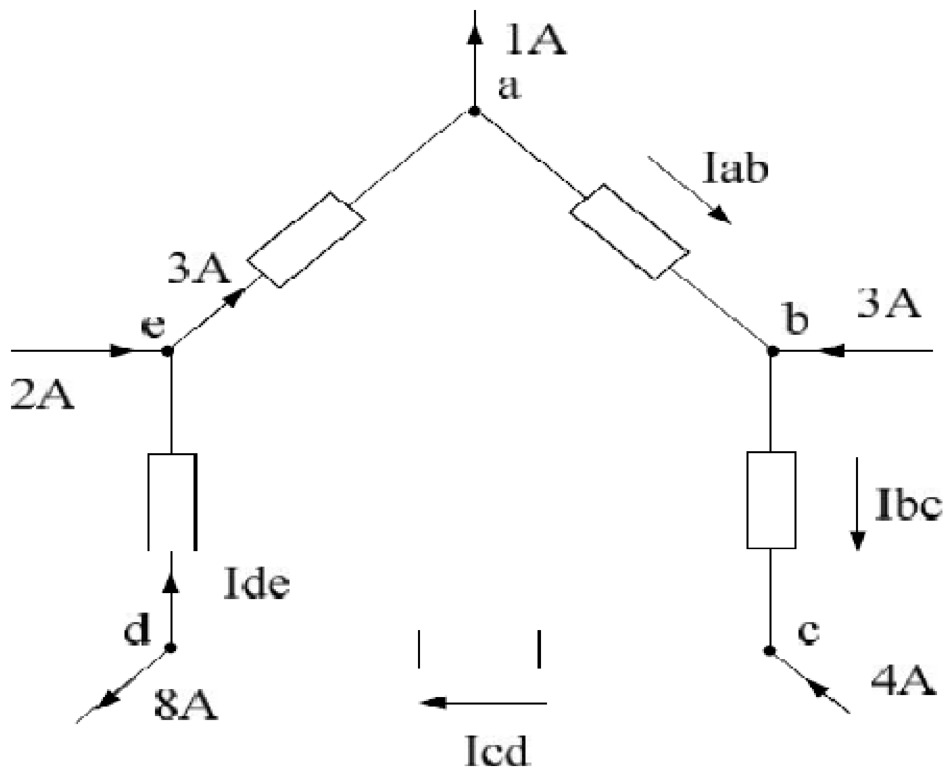
$$-I_1 + I_2 + I_3 = 0$$

$$13 + 5 - I = 0$$

$$18 - I = 0$$

$$\therefore I = 18\ A$$

Example :- Find the current in each section in the circuit. Shown ?



Solution :-

At node a

$$\mathbf{3 - 1 - I_{ab} = 0}$$

$$\mathbf{2 - I_{ab} = 0 \rightarrow I_{ab} = 2\ A}$$

At node b

$$\mathbf{I_{ab} + 3 - I_{bc} = 0}$$

$$\mathbf{2 + 3 - I_{bc} = 0 \rightarrow I_{bc} = 5\ A}$$

At node c

$$\mathbf{I_{bc} + 4 - I_{cd} = 0}$$

$$\mathbf{5 + 4 - I_{cd} = 0 \rightarrow I_{cd} = 9\ A}$$

At node d

$$I_{cd} - 8 - I_{de} = 0$$

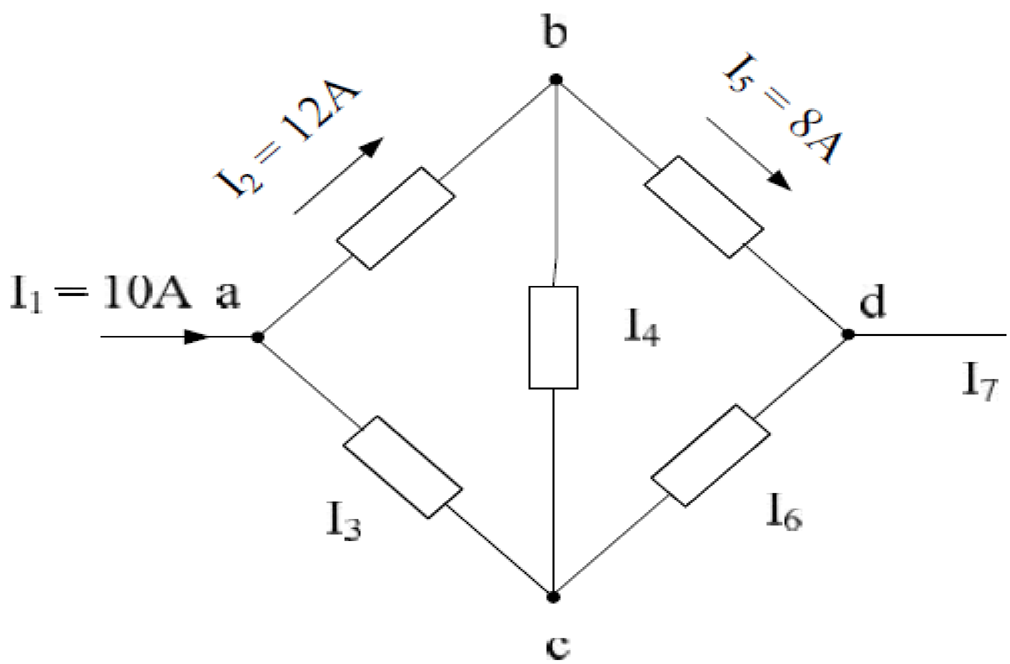
$$9 - 8 - I_{de} = 0 \rightarrow I_{de} = 1\text{ A}$$

At node e

$$I_{de} + 2 - 3 = 0$$

$$1 + 2 - 3 = 0 \rightarrow 0 = 0\ check$$

Example :- Find the magnitude and direction of the currents I_3 , I_4 , I_6 , I_7 in the following circuit Diagram?



Solution:

At node a ; suppose I_3 is entering

$$\boldsymbol{I_1 + I_3 - I_2 = 0}$$

$$\mathbf{10} + \boldsymbol{I_3} - \mathbf{12 = 0} \;\rightarrow \boldsymbol{I_3} = \mathbf{2\ A}$$

At node b;
I_2 enter , I_5 leave , ∴ I_4 must be leaving

$$\boldsymbol{I_2 = I_5 + I_4}$$

$$\mathbf{12 = 8} + \boldsymbol{I_4} \;\rightarrow \boldsymbol{I_4} = \mathbf{4\ A}$$

At node c;
I_4 enter , I_3 leave , ∴ I_6 leave

$$\boldsymbol{I_4 = I_3 + I_6}$$

$$\mathbf{4 = 2} + \boldsymbol{I_6} \;\rightarrow \boldsymbol{I_6} = \mathbf{2\ A}$$

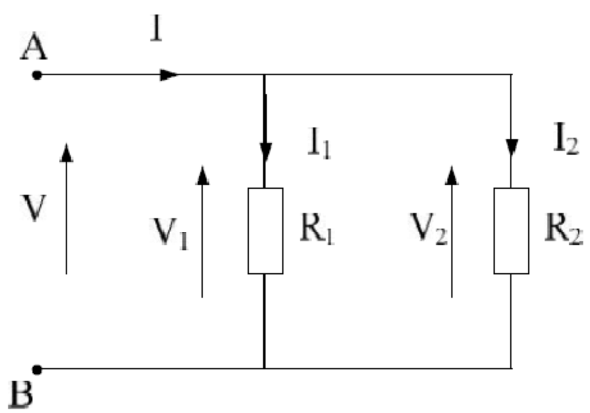
At node d;
I_5 and I_6 enter , I_7 leave

$$\boldsymbol{I_5 + I_6 = I_7}$$

$$\mathbf{8 + 2} = \boldsymbol{I_7} \;\rightarrow \boldsymbol{I_7} = \mathbf{10\ A}$$

Lecture 7

Resisters in Parallel :-



From K.V.L. $\quad V = V_1 = V_2$

From K.C.L. $\quad I = I_1 + I_2$

From Ω.L. $\quad I = \dfrac{V_1}{R_1} + \dfrac{V_2}{R_2}$

$$= V_1G_1 + V_2G_2$$

$$= V_1 (G_1 + G_2)$$

or $\quad I = V (G_1 + G_2)$

$$I = VG_T$$

Where $\quad G_T = G_1 + G_2$

Hence $\quad \dfrac{1}{R_T} = \dfrac{1}{R_1} + \dfrac{1}{R_2} = \dfrac{R_1 + R_2}{R_1 . R_2}$

or $\quad \boxed{R_T = \dfrac{R_1 . R_2}{R_1 + R_2}}$

In the same minner , if we have three resistors in parallel , then:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_T} = \frac{R_2.R_3 + R_1.R_3 + R_1.R_2}{R_1.R_2.R_3}$$

$$R_T = \frac{R_1.R_2.R_3}{R_2.R_3 + R_1.R_3 + R_1.R_2}$$

And , if we have N of parallel resistance , then

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ------\frac{1}{R_N}$$

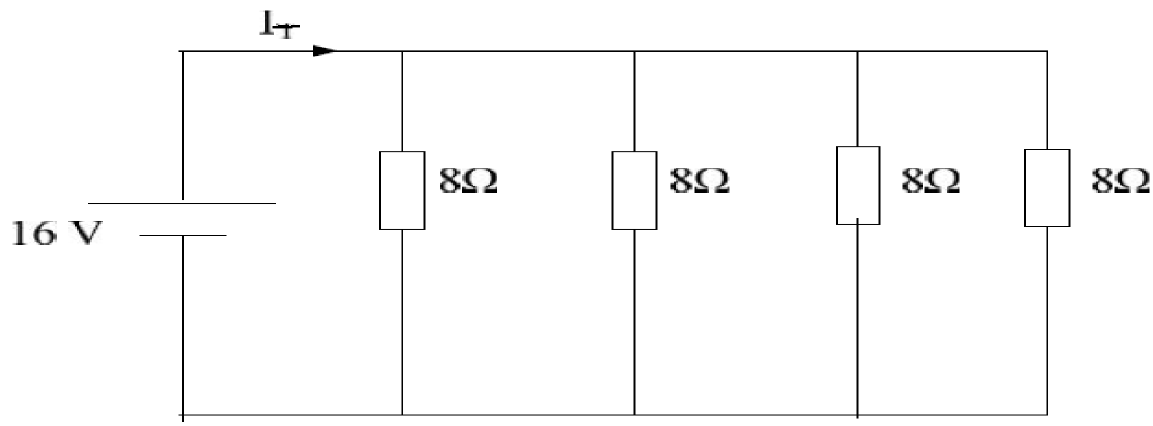
Also

$P_T = P_1 + P_2 + P_3$

$$P_1 = V_1 I_1 = I_1^2 R_1 = \frac{V_1^2}{R_1}$$

Source power $P_s = EI_T = I_T^2 R_T = \frac{E_T^2}{R_T}$

Example :- For the following circuit Find R_T , P_T , I_T , I_b?



Solution:

In case of equal resistors

$$R_T = \frac{R}{N} = \frac{8}{4} = 2\ \Omega$$

$$I_T = \frac{E}{R_T} = \frac{16}{2} = 8\ \mathrm{A}$$

$$I_{branch} = \frac{E}{R_1} = \frac{16}{8} = 2A$$

$$P_T = I_T^2 R_T = (8)^2.(2) = 128W$$

or $\quad P_T = E.I_T = 16 * 8 = 128W$

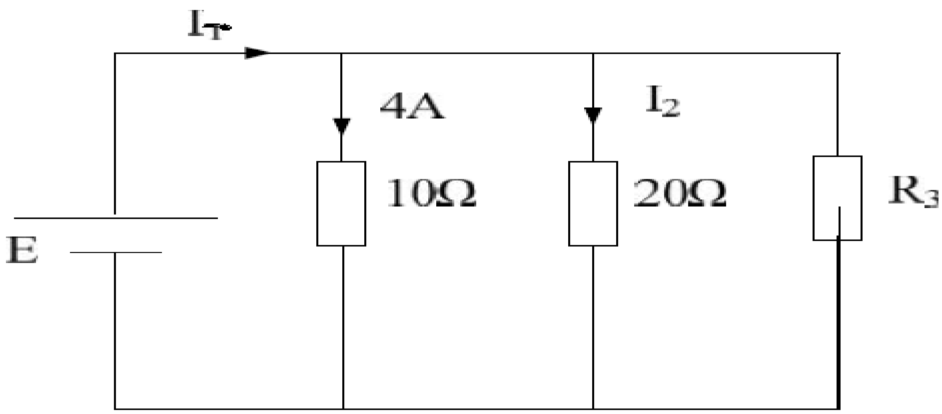
or $\quad P_T = P_1 + P_2 + P_3 + P_4$

$$= (2)^2 * 8 + (2)^2 * 8 + (2)^2 * 8 + (2)^2 * 8$$

$$= 32 + 32 + 32 + 32 = 128W$$

Example :- For the parallel network in figure below , find :-

a) R_3 , b) E , c) I_T , I_2 , d) P_2 ; given that $R_T = 4\ \Omega$?



Solution:

a)

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{4} = \frac{1}{10} + \frac{1}{20} + \frac{1}{R_3}$$

$$0.25 = 0.1 + 0.05 + \frac{1}{R_3}$$

$$0.25 - 0.1 - 0.05 = \frac{1}{R_3}$$

$$0.1 = \frac{1}{R_3} \Rightarrow R_3 = \frac{1}{0.1} = 10\Omega$$

b) $E = V_1 = I_1 R_1 = 4 * 10 = 40\ V$

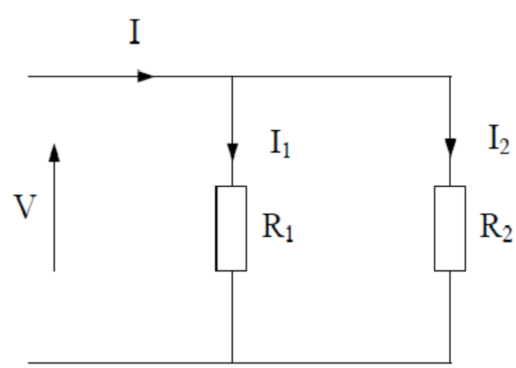
c) $$I_T = \frac{E}{R_T} = \frac{40}{4} = 10A$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{40}{20} = 2A$$

d) $$P_2 = I_2^2 R_2 = (2)^2 .(20) = 80W$$

or $$P_2 = \frac{V_2^2}{R_2}$$, or $P_2 = I_2 V_2$

Current division Rule :-



$$V = I\frac{R_1.R_2}{R_1 + R_2}$$

$$I_1 = \frac{V}{R_1} = \frac{I\frac{R_1.R_2}{R_1 + R_2}}{R_1}$$

$$\therefore I_1 = I\frac{R_2}{R_1 + R_2}$$

In the same manner

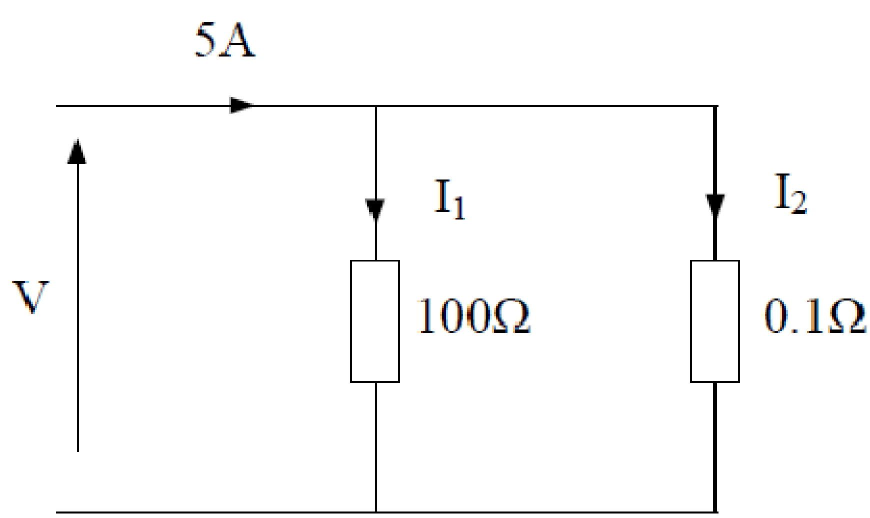
In the same miner

$$I_2 = I\frac{R_1}{R_1 + R_2}$$

Also $$\frac{I_1}{I_2} = \frac{I\dfrac{R_2}{R_1 + R_2}}{I\dfrac{R_1}{R_1 + R_2}} = \frac{R_2}{R_1}$$

$$\therefore \frac{I_1}{I_2} = \frac{R_2}{R_1} = \frac{G_1}{G_2}$$

Example :- For the following circuit. , find V , I_1 and I_2?



Solution:

$$R_T = \frac{R_1 . R_2}{R_1 + R_2} = \frac{100 \times 0.1}{100 + 0.1} = 0.0999\ \Omega$$

$$V = I . R_T = 5 \times\ 0.0999 = 0.4995\ \text{V}$$

Using current division Rule

$$I_1 = I . \frac{R_2}{R_1 + R_2} = 5 . \frac{0.1}{100 + 0.1} = 0.004995\ \text{A}$$

$$I_2 = I . \frac{R_1}{R_1 + R_2} = 5 . \frac{100}{100 + 0.1} = 4.995\ \text{A}$$

Or

$$I_1 = \frac{V}{R_1} = \frac{0.4995}{100} = 0.004995\ \text{A}$$

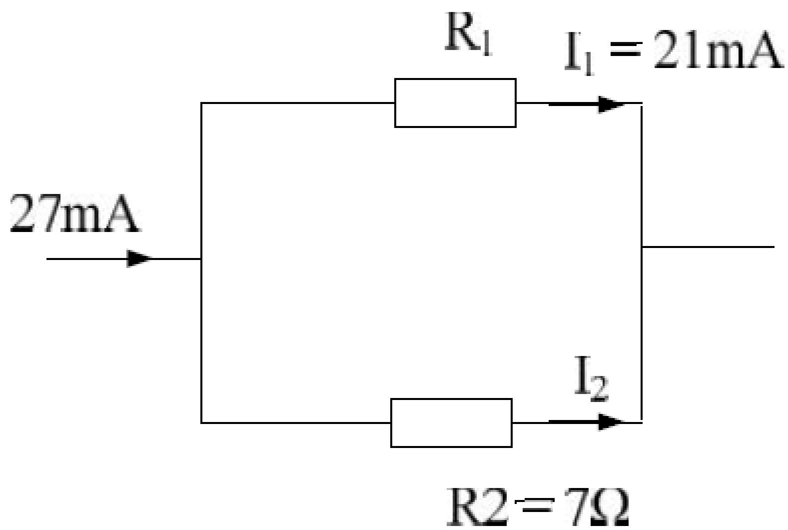
$$I_2 = \frac{V}{R_2} = \frac{0.4995}{0.1} = 4.995\ \text{A}$$

To check $I = I_1 + I_2$

$$5 = 0.004995 + 4.995$$

$$5 = 5\ ok$$

Example :- Determine the resistance R_1 in the circuit below?



Solution:

Using current division Rule

$$I_1 = I.\frac{R_2}{R_1+R_2} \rightarrow 21 \times 10^{-3} = \frac{27 \times 10^{-3} \times 7}{R_1 + 7}$$

$R_1 = 2\ \Omega$

Or

$I = I_1 + I_2 \ \rightarrow I_2 = I - I_1 = 27 \times 10^{-3} - 21 \times 10^{-3} = 6 \times 10^{-3}\ \mathrm{A} = 6\ \mathrm{mA}$

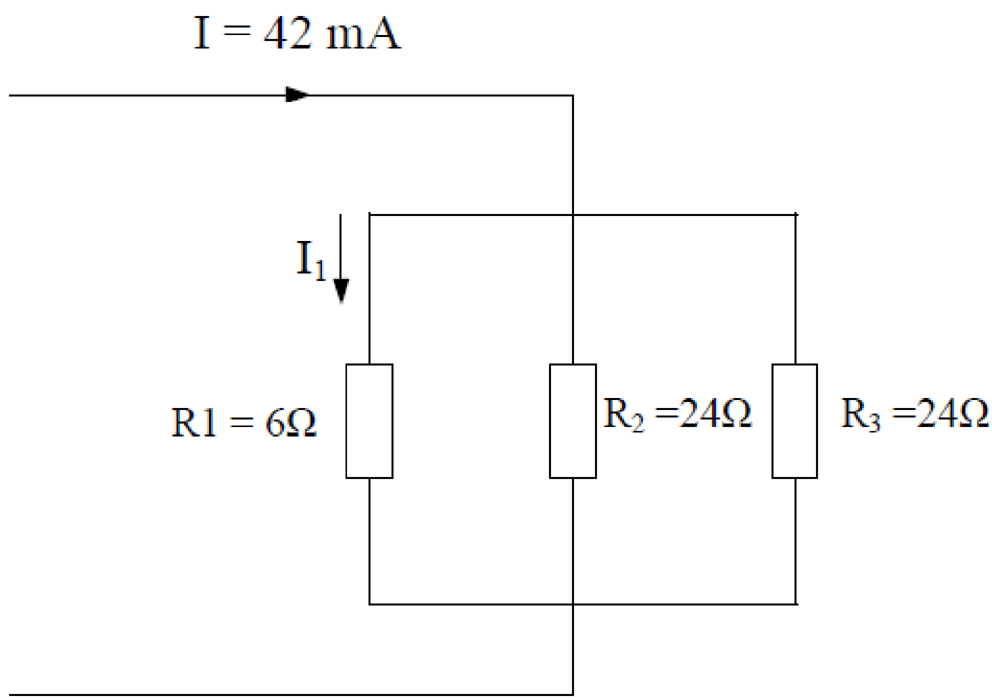
$V_2 = I_2.R_2 = 6 \times 10^{-3} \times\ 7 = 42\ \mathrm{mV}$

$V_1 = V_2 = 42\ \mathrm{mV}$

$$R_1 = \frac{V_1}{I_1} = \frac{42 \times 10^{-3}}{21 \times 10^{-3}} = 2\ \Omega$$

Lecture 8

Example :- Find the current I_1 , for the network shown:



Solution :- All resistance in parallel , so if we define that $R = R_2 // R_3$ then :-

$$R = \frac{R_2 R_3}{R_2 + R_3} = \frac{24 * 24}{24 + 24} = 12\Omega$$

Hence

$$I_1 = I\frac{R}{R + R_1} = \left(42 * 10^{-3}\right)\frac{12}{12 + 6} = 28mA$$

Voltage Regulation :-

Voltage Regulation $$V_R\% = \frac{V_{NL}-V_{FL}}{V_{FL}} \times 100\%$$

Where

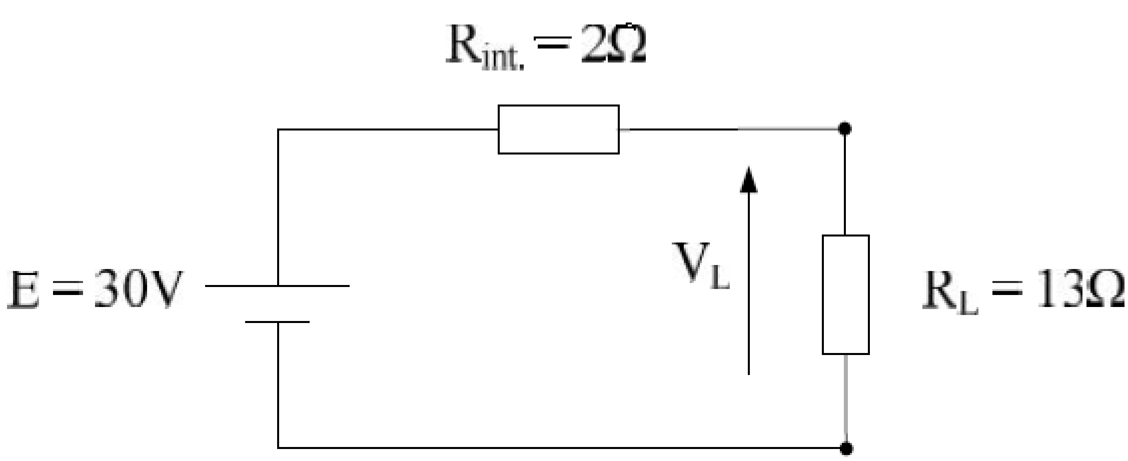
V_{NL} = No load voltage
V_{FL} = Full load voltage

Also we can write

$$V_R\% = \frac{R_{int}}{R_L} \times 100\%$$

Where
$R_{int.}$ = Internal resistor.
R_L = load resistor.

Example :- Find the voltage V_L and power lost to the internal resistance , if the applied load is 13 Ω , also find the voltage regulation ?



Solution:

$$I_L = \frac{E}{R_{int} + R_L} = \frac{30}{2 + 13} = 2A$$

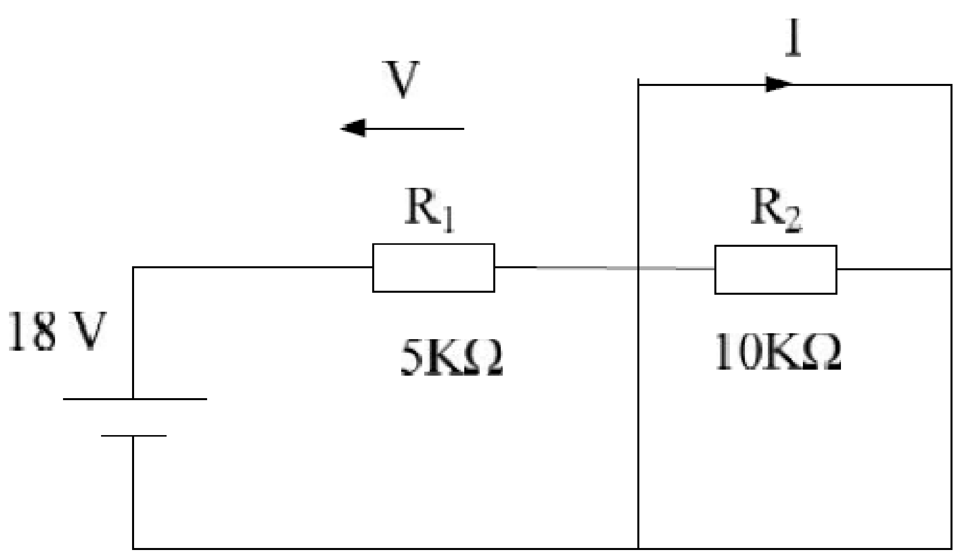
$$V_L = E - I_L R_{int.} = 30 - 2 * 2 = 26V$$

$$P_{loss} = I_L^2 R_{int} = (2)^2.(2) = 8W$$

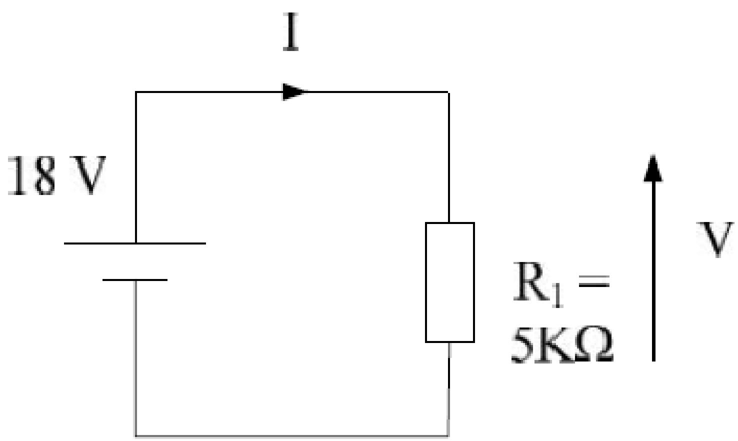
$$V_R\% = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100\% = \frac{30 - 26}{26} \times 100\% = 15.385\%$$

or $$V_R\% = \frac{R_{int}}{R_L} \times 100\% = \frac{2}{13} \times 100\% = 15.385\%$$

Example :- Calculate I & V for the network shown



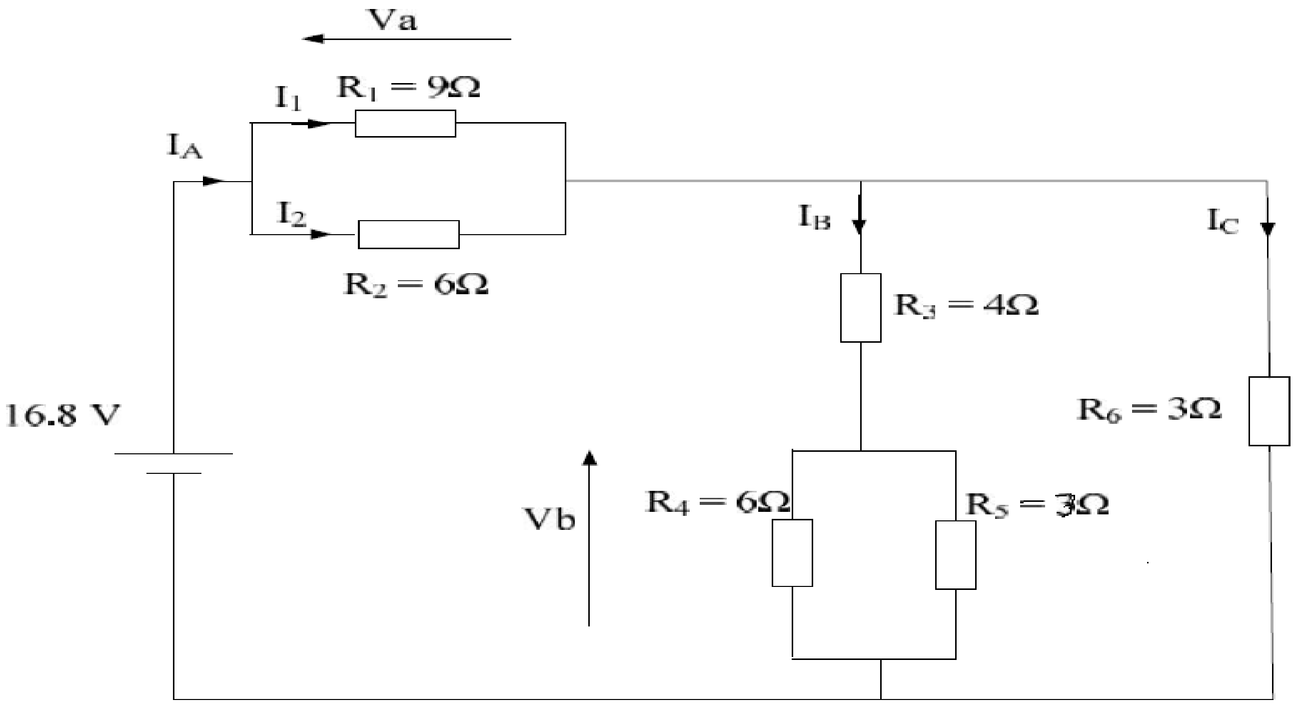
Solution :- We have a short circuit on R_2 resistance , hence no current through R_2 , hence the above circuit. Can redrawn as follows:



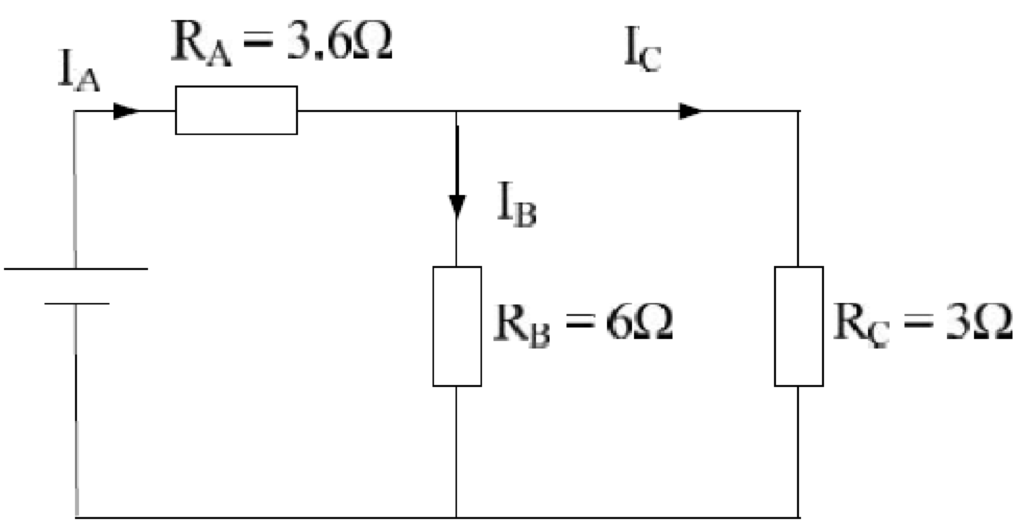
$$I = \frac{E}{R_T} = \frac{E}{R_1} = \frac{18}{5 \times 10^3} = 3.6 \text{ mA}$$

$$V = I.R_1 = 0.0036 \times 5 \times 10^3 = 18 \text{ V} = E$$

Example :- For the following circuit Network , find R_T , I_A , I_B , I_C , I_1 , I_2 , V_a , V_b ,?



Solution:



$$R_A = \frac{R_1 R_2}{R_1 + R_2} = \frac{9 * 6}{9 + 6} = 3.6\Omega$$

$$R_B = R_3 + \frac{R_4 \times R_5}{R_4 + R_5} = 4 + \frac{6 \times 3}{6 + 3} = 6\ \Omega$$

$$R_C = 3\ \Omega$$

$$R_T = R_A + \frac{R_B \times R_C}{R_B + R_C} = 3.6 + \frac{6 \times 3}{6 + 3} = 5.6\ \Omega$$

$$I_A = \frac{E}{R_T} = \frac{16.8}{5.6} = 3\ \text{A}$$

Apply C.D.R

$$I_B = I_A \times \frac{R_C}{R_B + R_C} = 3 \times \frac{3}{3 + 6} = 1\ \text{A}$$

By K.C.L.

$$I_C = I_A - I_B = 3 - 1 = 2\ \text{A}$$

$$I_1 = I_A \times \frac{R_2}{R_2 + R_1} = 3 \times \frac{6}{6 + 9} = 1.2\ \text{A}$$

$$I_2 = I_A \times \frac{R_1}{R_1 + R_2} = 3 \times \frac{9}{9 + 6} = 1.8\ \text{A}$$

Or

$$I_2 = I_A - I_1 = 3 - 1.2 = 1.8\ \text{A}$$

$$V_a = I_1 R_1 = 1.2 \times 9 = 10.8\ \text{V}$$

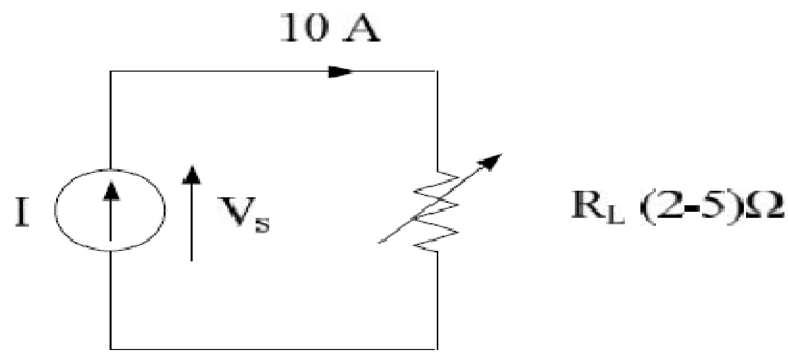
$$I_4 = I_B \times \frac{R_5}{R_5 + R_4} = 1 \times \frac{3}{3 + 6} = 0.3333\ \text{A}$$

$$V_b = I_4 R_4 = 0.3333 \times 6 = 2 \text{ V}$$

Lecture 9

Current Source :-

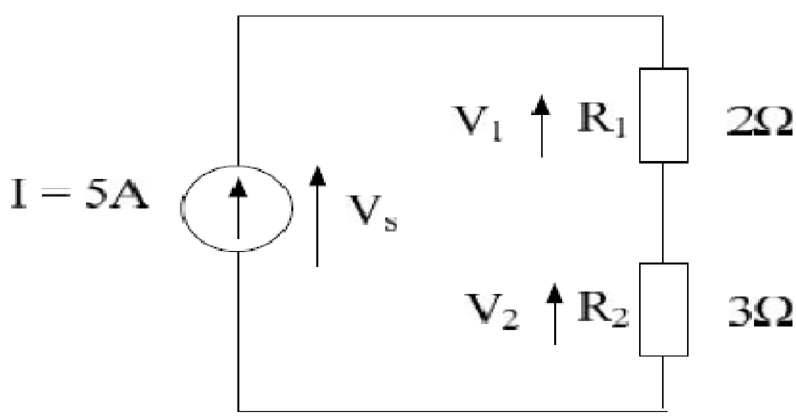
Example :- Find the voltage (V_s) for the circuit below:



Solution :-

$V_s = IR_L = 10 * 2 = 20\ V$ if $R_L = 2\ \Omega$
$V_s = IR_L = 10 * 5 = 50\ V$ if $R_L = 5\ \Omega$

Example :- Calculate V_1 , V_2 , V_s for the following circuit:

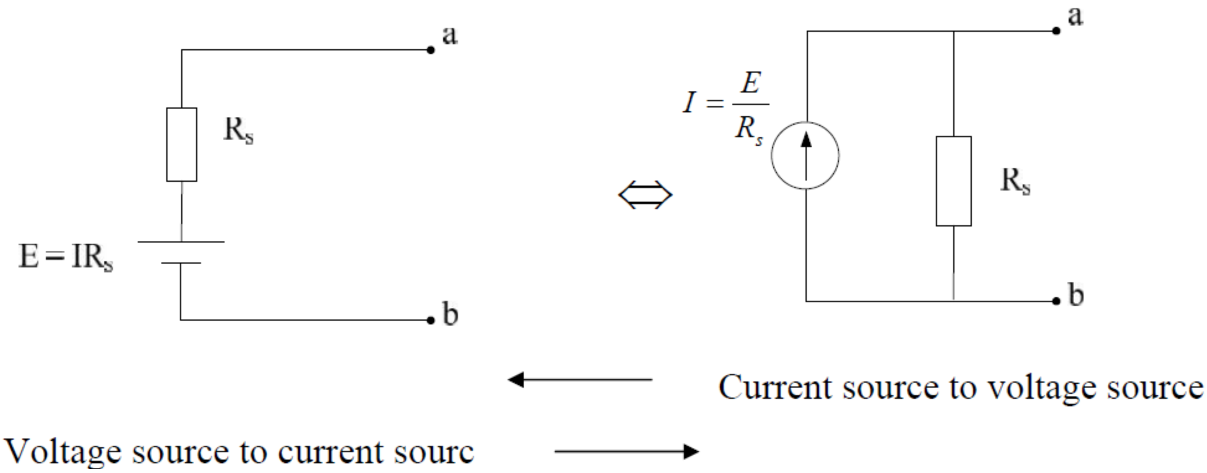


Solution :-

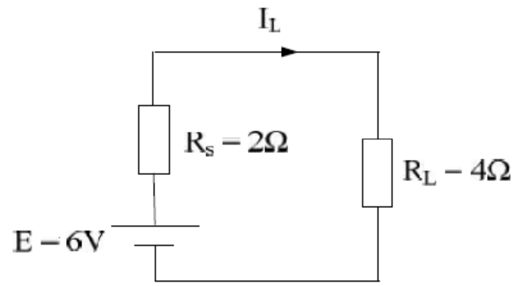
$V_1 = IR_1 = 5 * 2 = 10$ V
$V_2 = IR_2 = 5 * 3 = 15$ V
$V_s = V_1 + V_2 = 10 + 15 = 25$ V

Source Conversions :-

A voltage source with voltage E and series resistor R_s can be replaced by a current source with a current I and parallel resistor R_s as shown :-



Example :- Convert the voltage source in the cct. Below to a current source, then calculate the current through the load for each source:



Solution :-

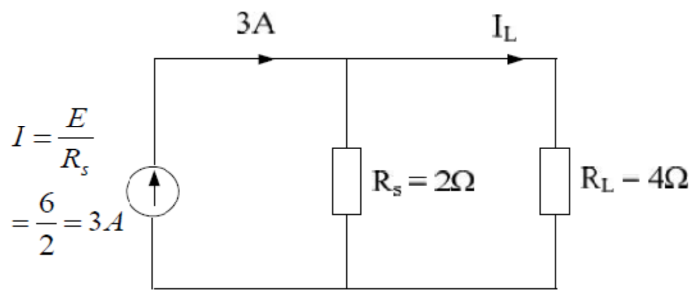
$$I_L = \frac{E}{R_s + R_L} = \frac{6}{2+4} = 1A$$

⇓

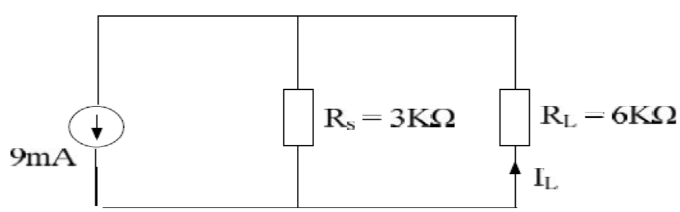
- For the current source cct.

$$I_L = I\frac{R_s}{R_s + R_L} = 3\left(\frac{2}{2+4}\right) = 1A$$

لاحظ ان I_L متساوي في الحالتين و هذا صحيح .



Example :- Convert the current source in the cct. Shown below to a voltage source and determine I_L for each cct.:



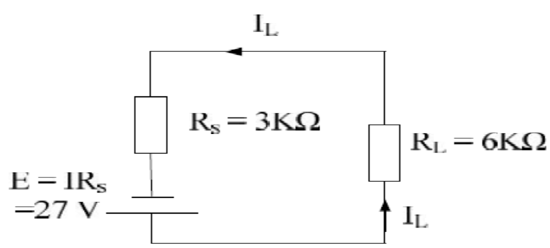
Solution :-

- For the current cct.

$$I_L = I\frac{R_s}{R_s + R_L} = (9*10^{-3})\left(\frac{3*10^3}{3*10^3 + 6*10^3}\right)$$

$$\therefore I_L = 3mA$$

⇓

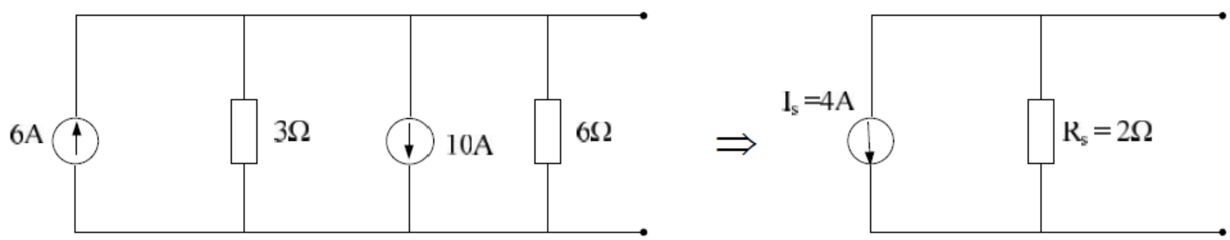


- For the voltage source cct.

$$I_L = \frac{E}{R_T} = \frac{E}{R_s + R_L} = \frac{27}{(3+6)*10^3}$$

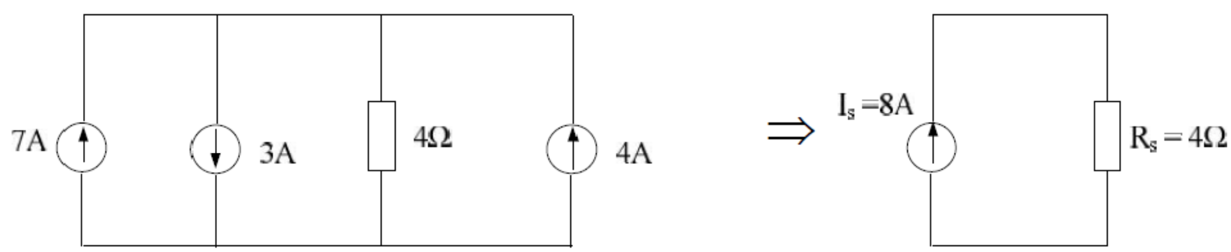
$$\therefore I_L = 3mA$$

Current source in parallel :-



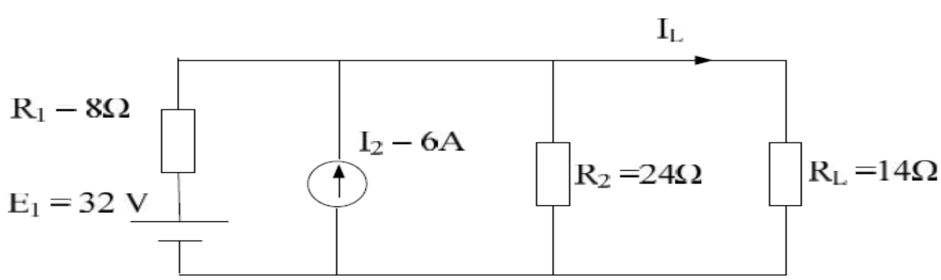
$I_s = 10 - 6 = 4$ A   & $R_s = 3\ \Omega \,//\, 6\ \Omega = 2\ \Omega$

Example :-

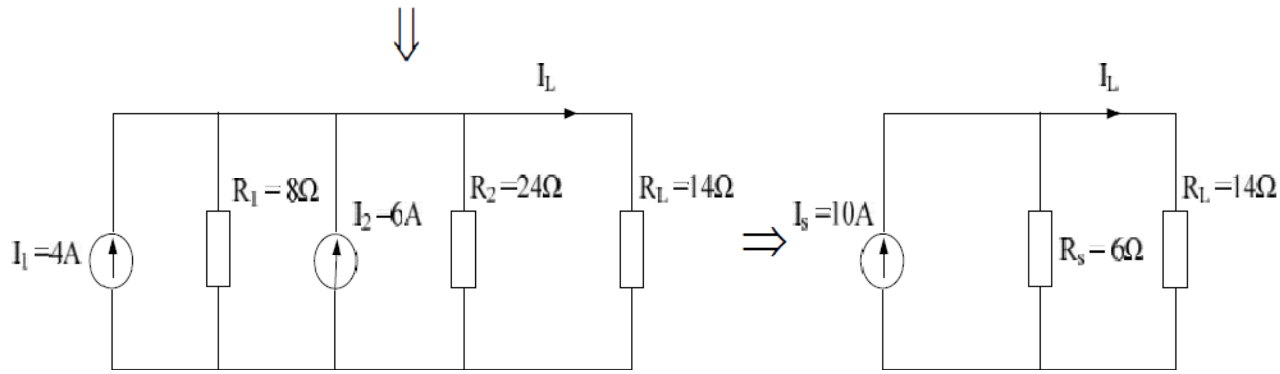


$I_s = 7 - 3 + 4 = 8$ A

Example :- Find the load current in the following cct.:



Solution :-



$$I_1 = \frac{E}{R_1} = \frac{32}{8} = 4A$$

$$Is = I_1 + I_2 = 4 + 6 = 10\ A$$

$$R_s = R_1 \,//\, R_2 \;= \frac{8 * 24}{8 + 24} = 6\Omega \Rightarrow \therefore I_L = I_s \frac{R_s}{R_s + R_L} = \frac{10 * 6}{6 + 14} = 3A$$

Lecture 10

Matrices :-

Second order determinate

$$D = \begin{array}{cc} \underline{\text{Col. 1}} & \underline{\text{Col. 2}} \\ \multicolumn{2}{c}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \end{array} = a_1 b_2 - a_2 b_1$$

Col. 1	Col. 2		Col. 3
a_1x	b_1y	—	c_1
a_2x	b_2y	—	c_2

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$x = \frac{D_1}{D} = \frac{\begin{bmatrix} c_1 & b_1 \\ c_2 & b_2 \end{bmatrix}}{\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}} = \frac{c_1 b_2 - b_1 c_2}{a_1 b_2 - b_1 a_2}$$

$$y = \frac{D_2}{D} = \frac{\begin{bmatrix} a_1 & c_1 \\ a_2 & c_2 \end{bmatrix}}{\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}} = \frac{a_1 c_2 - c_1 a_2}{a_1 b_2 - b_1 a_2}$$

Example :- Find the value of D

$$D = \begin{bmatrix} 4 & -1 \\ 6 & 2 \end{bmatrix}$$

Solution :-

$$D = \begin{bmatrix} 4 & -1 \\ 6 & 2 \end{bmatrix} = 4 * 2 - (-1) * 6 = 8 + 6 = 14$$

Example :- Solving the equations below by determinates

$$4I_1 - 6\,I_2 = 8$$

$$2I_1 + 4\,I_2 = 20$$

Solution :-

$$\begin{bmatrix} 4 & -6 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 8 \\ 20 \end{bmatrix}$$

$$I_1 = \frac{D_1}{D} = \frac{\begin{bmatrix} 8 & -6 \\ 20 & 4 \end{bmatrix}}{\begin{bmatrix} 4 & -6 \\ 2 & 4 \end{bmatrix}} = \frac{8 * 4 - (-6) * 20}{4 * 4 - (-6) * 2} = \frac{32 + 120}{16 + 12} = 5.428A$$

$$I_1 = \frac{D_1}{D} = \frac{\begin{bmatrix} 4 & 8 \\ 2 & 20 \end{bmatrix}}{28} = \frac{4 * 20 - 8 * 2}{28} = 2.28A$$

<u>Third – order determinant :-</u>

$$D = \left|\begin{array}{ccc|cc} a_1 & b_1 & c_1 & a_1 & b_1 \\ a_2 & b_2 & c_2 & a_2 & b_2 \\ a_3 & b_3 & c_3 & a_3 & b_3 \end{array}\right.$$

$$D = \left[a_1 b_2 c_3 + a_1 c_2 a_3 + c_1 a_2 b_3\right] - \left[c_1 b_2 a_3 + a_1 c_2 b_3 + b_1 a_2 c_3\right]$$

Example :- Find the value of D

$$D = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 1 & 0 \\ 0 & 4 & 2 \end{bmatrix}$$

Solution :-

$$D = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 1 & 0 \\ 0 & 4 & 2 \end{bmatrix} \begin{matrix} 1 & 2 \\ -2 & 1 \\ 0 & 4 \end{matrix}$$

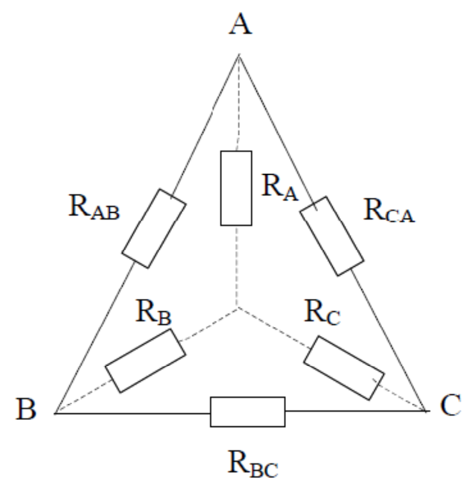
$$\therefore D = [1*1*2 + 2*0*0 + 3*(-2)*4] - [0*1*3 + 4*0*1 + 2*-2*2]$$

$$D = [2 + 0 - 24] - [0 + 0 + 8] = -22 + 8 = -14$$

Star – Delta ($Y \rightarrow \Delta$) and Delta – Star ($\Delta \rightarrow Y$) transformation :-

1.) Delta – Star ($\Delta \rightarrow Y$) transformation :-

If the value of R_{AB} , R_{CA} , R_{BC} are known, and we need to get the values of R_A , R_B , R_C ; then :-



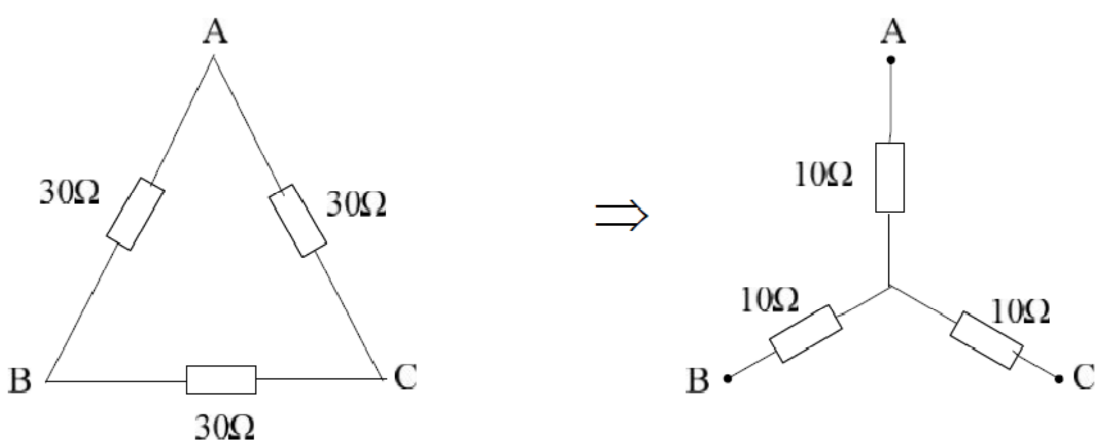
$$R_A = \frac{R_{AB} R_{CA}}{R_{AB} + R_{CA} + R_{BC}}$$

$$R_B = \frac{R_{AB} R_{BC}}{R_{AB} + R_{CA} + R_{BC}}$$

$$R_C = \frac{R_{CA} R_{BC}}{R_{AB} + R_{CA} + R_{BC}}$$

If $R_{Ab} = R_{BC} = R_{CA} = R_\Delta$, in this case $R_A = R_B = R_C = \frac{R_\Delta}{3} = R_Y$

or $R_Y = \frac{R_\Delta}{3}$



Lecture 11

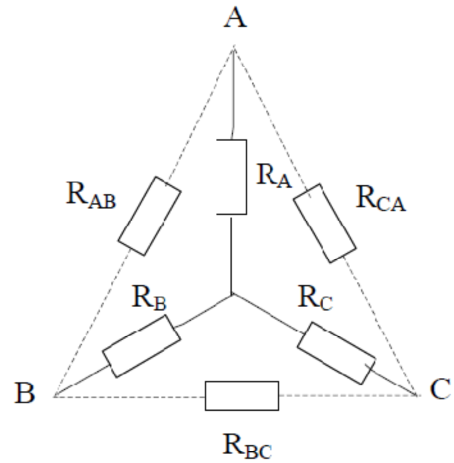
2.) Star – Delta ($Y \rightarrow \Delta$) transformation :-

If the value of R_A , R_B , R_C are known , and we need to get the values of R_{AB} , R_{CA} , R_{BC} ; as follows :-

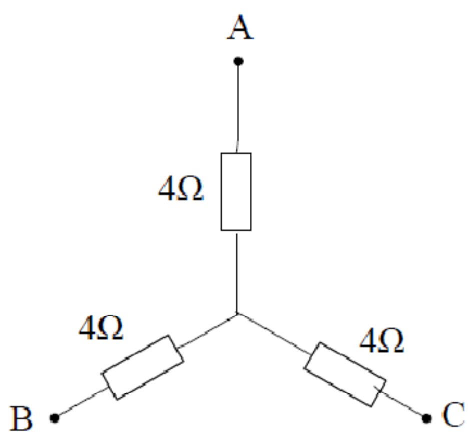
$$R_{AB} = \frac{R_A R_B + R_B R_C + R_A R_C}{R_C}$$

$$R_{BC} = \frac{R_A R_B + R_B R_C + R_A R_C}{R_A}$$

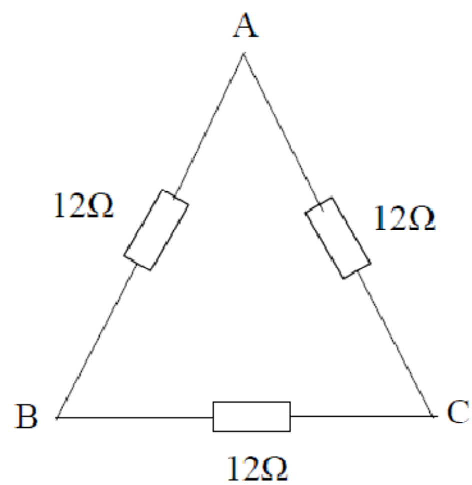
$$R_{CA} = \frac{R_A R_B + R_B R_C + R_A R_C}{R_B}$$



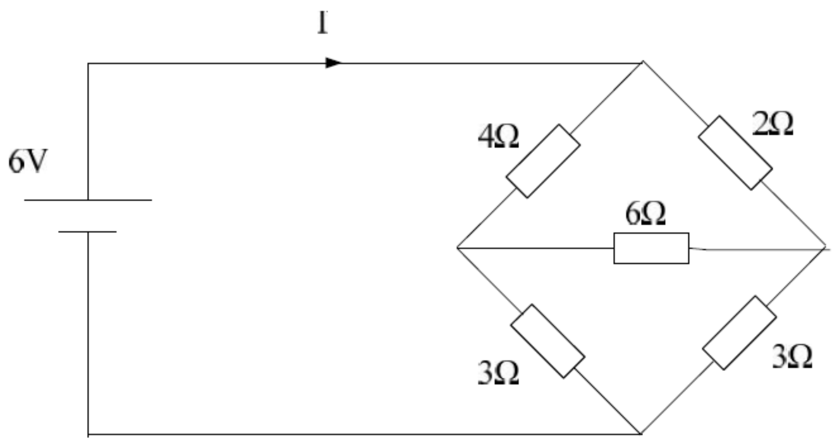
If $R_A = R_B = R_C = R_Y$, in this case $R_{AB} = R_{CA} = R_{BC} = R_\Delta = 3\ R_Y$



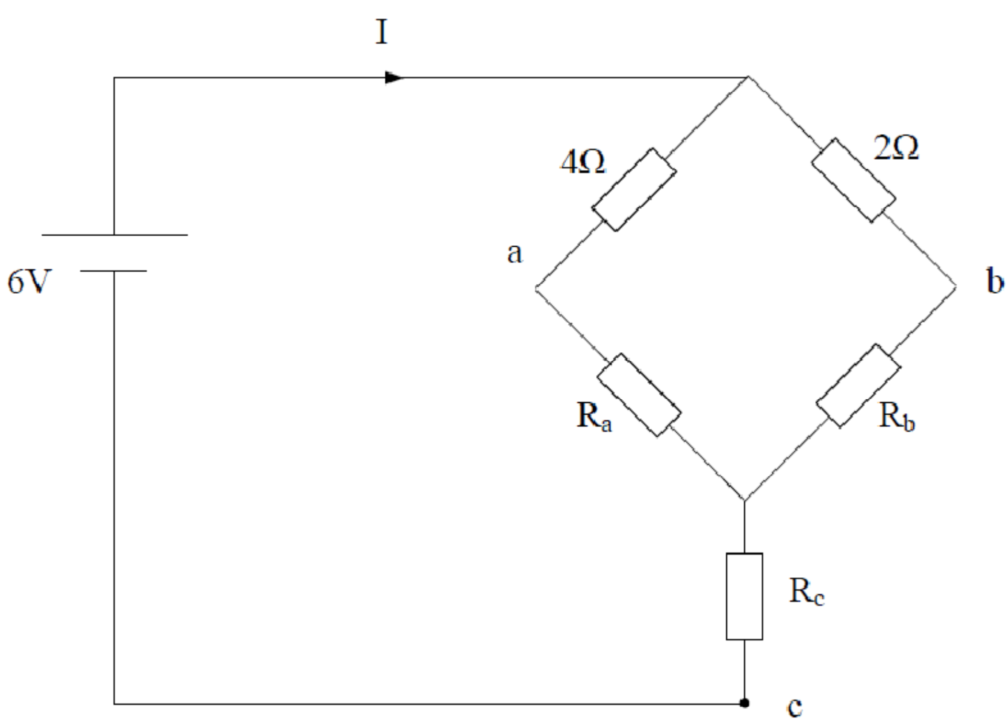
$\Rightarrow$



Example :- For the following network , find I ?



Solution :- The resistances (6 , 3 , 3) are delta , can convert to star connection as follows:

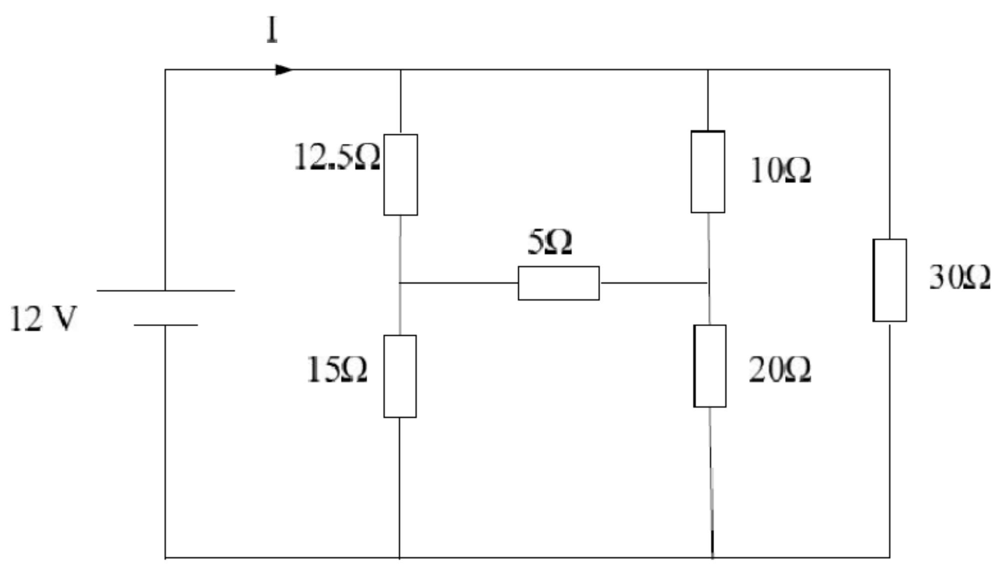


$$R_a = \frac{6*3}{6+3+3} = 1.5\Omega \quad , \; R_b = \frac{6*3}{6+3+3} = 1.5\Omega \quad , \quad R_c = \frac{3*3}{6+3+3} = 0.75\Omega$$

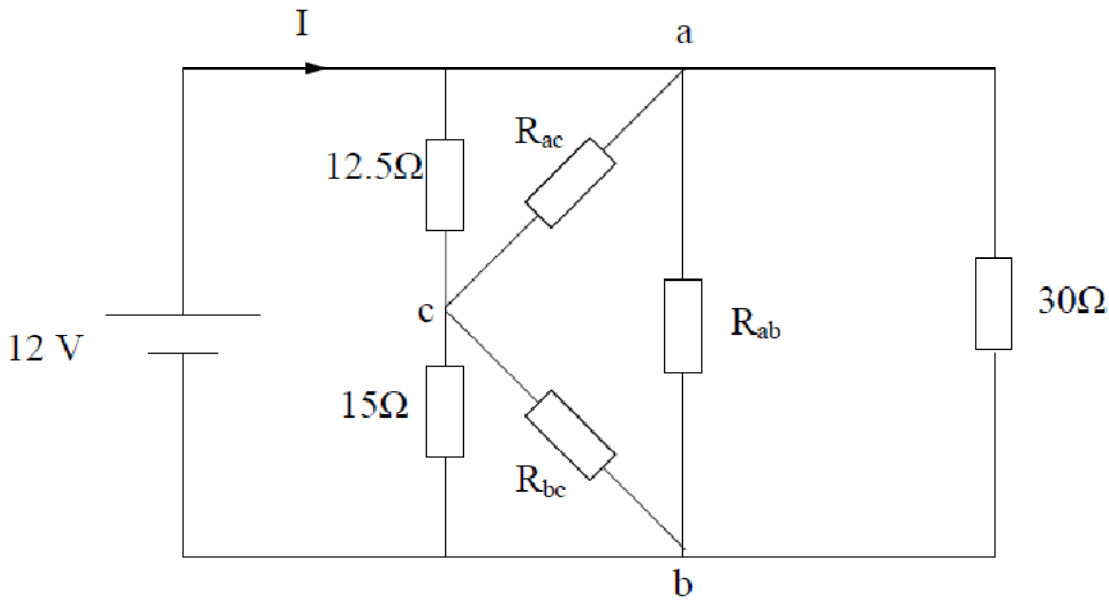
$$R_T = [(4+R_a)//(2+R_b)] + R_c$$
$$= [(5.5)//(3.5)] + R_c$$
$$= \left[\frac{5.5*3.5}{5.5+3.5}\right] + 0.75$$

$$I = \frac{E}{R_T} = \frac{6}{2.889} = 2.077A$$

Example :- Find I for the following cct. network :-



Solution :- The resistors (5Ω , 10Ω , 20Ω) are star convert to delta

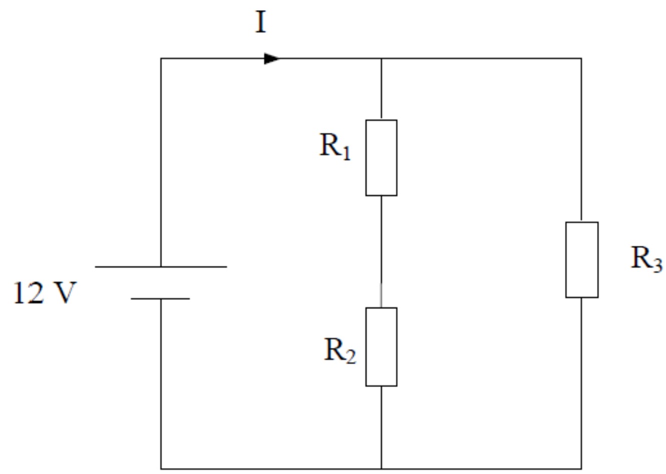


$$R_{ab} = \frac{10*20+10*5+5*20}{5} = 70\Omega$$

$$R_{ac} = \frac{10*20+10*5+5*20}{20} = 17.5\Omega$$

$$R_{bc} = \frac{10*20+10*5+5*20}{10} = 35\Omega$$

It is clear that (12.5 Ω // R_{ac}) and (15 Ω // R_{bc}) and (30 Ω // R_{ab}) , hence the circuit can be reduce to the following network :-



$$R_1 = (12.5 // R_{ac}) = \frac{12.5 * 17.5}{12.5 + 17.5} = 7.3\Omega$$

$$R_2 = (15 // R_{bc}) = \frac{15 * 35}{15 + 35} = 10.5\Omega$$

$$R_3 = (30 // R_{ab}) = \frac{30 * 70}{30 + 70} = 21\Omega$$

$$\therefore R_T = (R_1 + R_2) // R_3$$

$$= (7.3 + 10.5) // 21$$

$$= 17.8 // 21 = \frac{17.8 * 21}{17.8 + 21} = 9.634\Omega$$

$$I = \frac{E}{R_T} = \frac{12}{9.634} = 1.246A$$

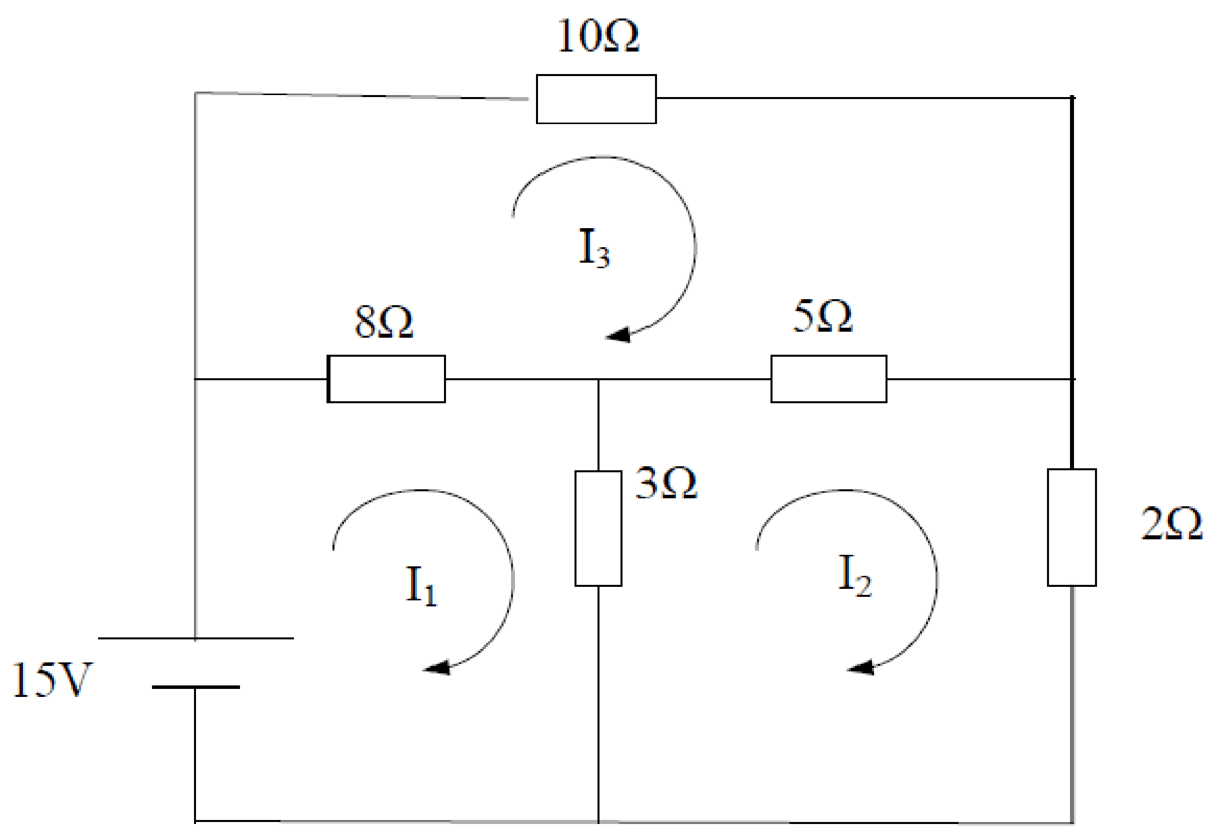
Lecture 12

Network Solution :-

To solve a circuit is to find the current and voltage in all branches.

1) Loop (Mesh) current method :-

Example(1):- Find the current through the 10 Ω resistor of the network shown:



Solution :-

– مجموع المقاومات * تيار الـ Loop + المقاومة المشتركة * تيار الـ Loop المشترك ∓ الفولتية (حسب اتجاهها مع تيار الـ Loop) = صفر .

The loop equations are :-

Loop 1 :-

$$-(8+3)I_1 + 3I_2 + 8I_3 + 15 = 0$$

Loop 2 :-

$$-(3+5+2)I_2 + 3I_1 + 5I_3 = 0$$

Loop 3 :-

$$-(10+8+5)I_3 + 8I_1 + 5I_2 = 0$$

Rearrange the equations , then :-

$$-11I_1 + 3I_2 + 8I_3 = -15$$

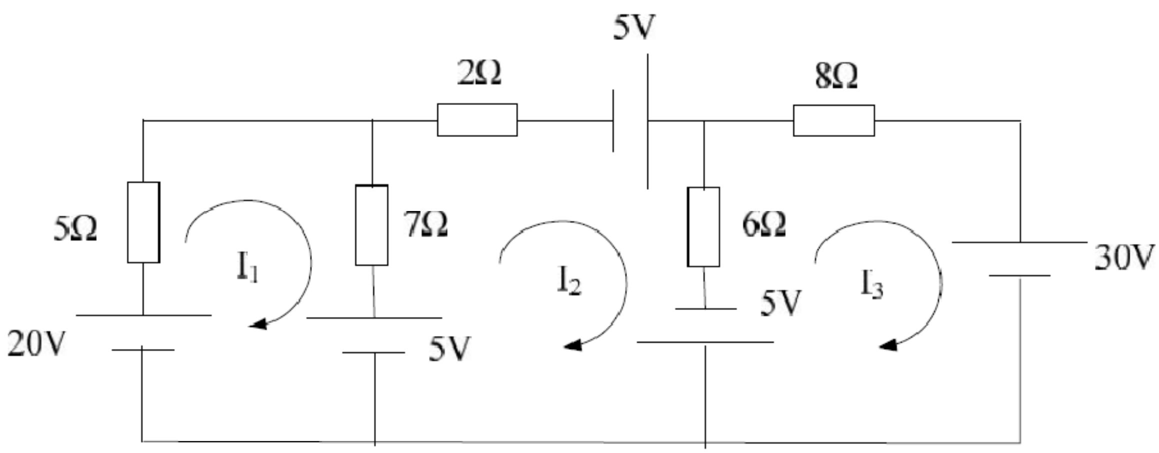
$$3I_1 - 10I_2 + 5I_3 = 0$$

$$8I_1 + 5I_2 - 23I_3 = 0$$

$$I_3 = \frac{D_3}{D} = \frac{\begin{bmatrix} -11 & 3 & -15 \\ 3 & -10 & 0 \\ 8 & 5 & 0 \end{bmatrix}}{\begin{bmatrix} -11 & 3 & 8 \\ 3 & -10 & 5 \\ 8 & 5 & -23 \end{bmatrix}} = 1.22A$$

$$\therefore I_3 = I_{10\Omega} = 1.22A$$

Example(2):- Solve following circuit diagram;



Solution :-

$$-I_1\,(\,5+7\,) + 7I_2 + 20 - 5 = 0$$

$$-I_2\,(\,7+2+6\,) + 7I_1 + 6I_3 + 5 + 5 + 5 = 0$$

$$-I_3\,(\,6+8\,) + 6I_2 - 5 - 30 = 0$$

Rearrange;

$-12I_1 + 7I_2 + 0 = -15$ ------------------- (1)

$7I_1 - 15I_2 + 6I_3 = -15$ ------------------- (2)

$0 + 6I_2 - 14I_3 = 35$ ------------------- (3)

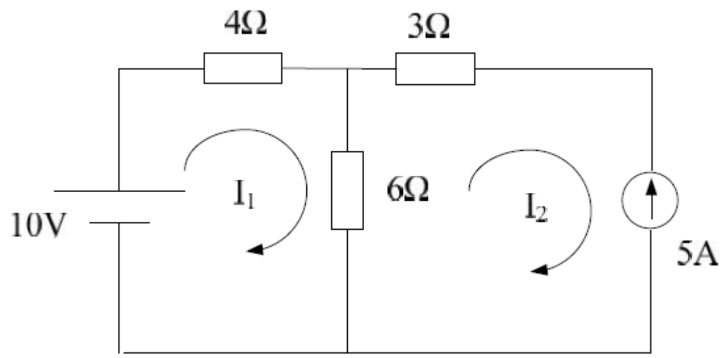
U

$$I_1 = \frac{D_1}{D} = \frac{\begin{bmatrix} -15 & 7 & 0 \\ -15 & -15 & 6 \\ 35 & 6 & -14 \end{bmatrix}}{\begin{bmatrix} -12 & 7 & 0 \\ 7 & -15 & 6 \\ 0 & 6 & -14 \end{bmatrix}} = \frac{2610}{1402} = 1.862A$$

$$I_2 = \frac{D_2}{D} = \frac{\begin{bmatrix} -12 & -15 & 0 \\ 7 & -15 & 6 \\ 0 & 35 & -14 \end{bmatrix}}{1402} = \frac{1470}{1402} = 1.049A$$

$$I_3 = \frac{D_3}{D} = \frac{\begin{bmatrix} -12 & 7 & -15 \\ 7 & -15 & -15 \\ 0 & 6 & 35 \end{bmatrix}}{1402} = \frac{-2875}{1402} = -2.05A$$

Example(3):- Find the current in the 10V source , for the following network;



Solution :-

$$I_2 = -5\ A$$

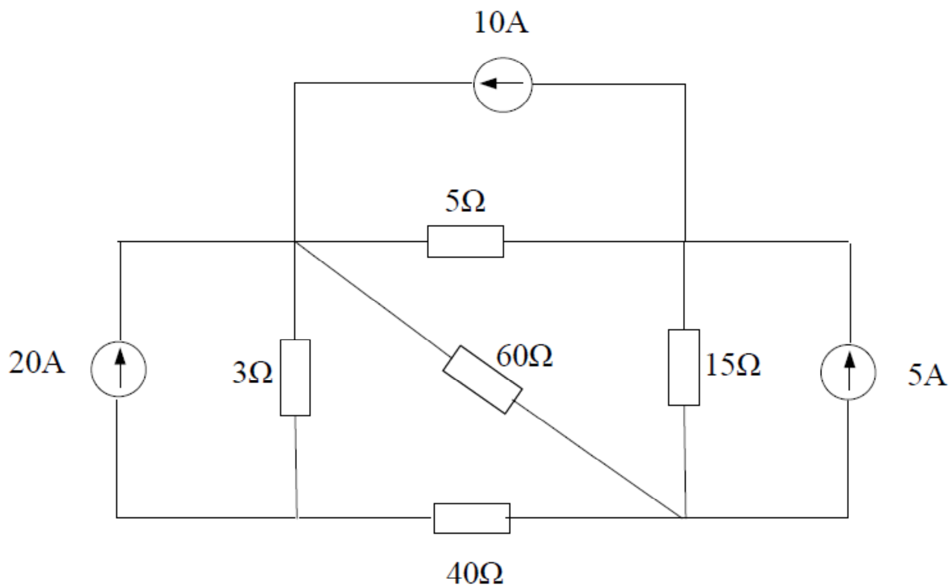
Hence , we need only one equation to solve this circuit

$$-I_1\ (\ 4+6\ ) + 6 * (\ -5\ ) + 10 = 0$$

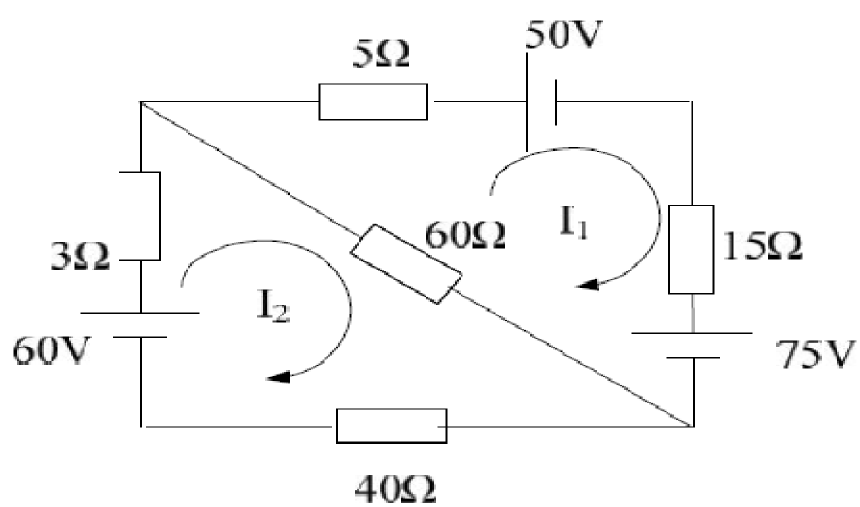
$$-10I_1 - 20 = 0 \rightarrow -10I_1 = 20$$

$$\therefore I_1 = \frac{20}{-10} = -2A$$

Example(5):- Solve the following circuit diagram .



Solution:- The above diagram can be reduced to the following diagram;



$$-I_1 (5+15+60) + 60I_2 - 50 - 75 = 0$$

$$-I_2 (3+60+40) + 60I_1 + 60 = 0$$

Rearrange:-

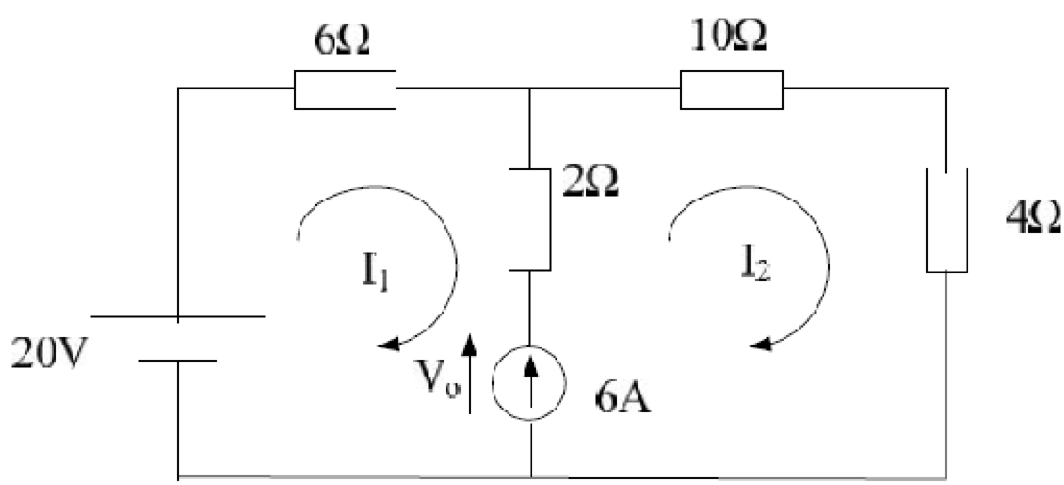
$$-80I_1 + 60I_2 = 125 \quad \text{------------------} \quad (1)$$

$$60I_1 - 103I_2 = -60 \quad \text{------------------} \quad (2)$$

$$\left.\begin{aligned} I_1 &= \frac{D_1}{D} \\ \\ I_2 &= \frac{D_2}{D} \end{aligned}\right\}$$

أكمل الحل

Example(6):- Solve the following circuit diagram:



Solution:-

$-(6+2)I_1 + 2I_2 + 20 - V_o = 0$ ------------------- (1)

$-(10+2+4)I_2 + 2I_1 + V_o = 0$ ------------------- (2)

$I_2 - I_1 = 6$ ------------------- (3)

Add eq. 1 & eq. 2

$-8I_1 + 2I_2 + 20 - 16\,I_2 + 2I_1 = 0$ ------------------- (1)

$-6I_1 - 14I_2 = -20$ ------------------- (2)

From eq. 3

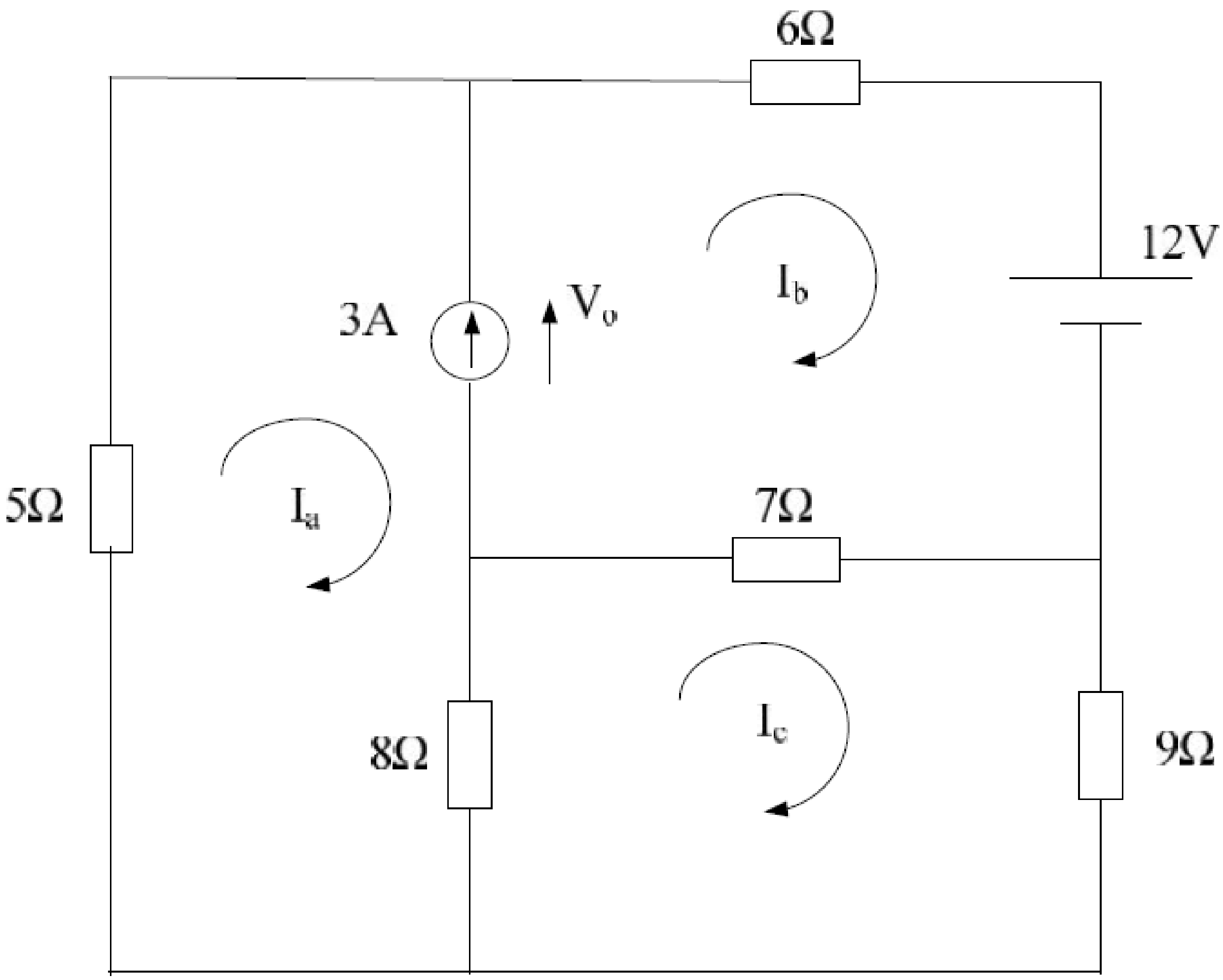
$I_1 - I_2 = -6$

$$I_1 = \frac{D_1}{D} = \frac{\begin{bmatrix} -20 & -14 \\ -6 & -1 \end{bmatrix}}{\begin{bmatrix} -6 & -14 \\ 1 & -1 \end{bmatrix}} = \frac{20-84}{6+14} = -3.2A$$

$$I_2 = \frac{D_2}{D} = \frac{\begin{bmatrix} -6 & -20 \\ 1 & -6 \end{bmatrix}}{20} = \frac{36+20}{20} = 2.8A$$

Lecture 13

Example (7) : Solve the following circuit , using loop current method :-



Solution:

Loop a :-

$-(5+8)I_a + 8I_c - V_o = 0$ ------------------- (1)

Loop b :-

$-(6+7)I_b + 7I_c + V_o - 12 = 0$ ------------------- (2)

Loop c :-

$-(8+7+9)I_c + 8I_a + 7I_b = 0$ ------------------ (3)

$I_b - I_a = 3$ ------------------ (4)

- ملاحظـة :- فـي مثـل هـذه الاسـئلة التـي تحتـوي علـى V_o ، نجـري عمليـات جمـع او طـرح المعادلات التي تحتوي على V_o للتخلص منها و نبسط الحل .

أمـا اذا كـان المطلـوب ايجـاد V_o ، فايضـا نقـوم بـالتخلص منهـا اولا (عـن طريـق جمـع او طـرح المعادلات التي تحتوي على V_o) ثم بعد ايجاد التيارات نعوضها في المعادلة التي تحتوي علـى V_o ونجدها .

Loop a+b :

$-13I_a - 13I_b + 15I_c - 12 = 0$ ------------------ (1′)

Loop c :-

$-24I_c + 8I_a + 7I_b = 0$ ------------------ (2′)

$I_b - I_a = 3$ ------------------ (3′)

Rearrange Eq.s :-

$-13I_a - 13I_b + 15I_c = 12$ ------------------ (1″)

$8I_a + 7I_b - 24I_c = 0$ ------------------ (2″)

$- I_a + I_b = 3$ ------------------ (3″)

$$I_a = \frac{D_1}{D} = \frac{\begin{bmatrix} 12 & -13 & 15 \\ 0 & 7 & -24 \\ 3 & 1 & 0 \end{bmatrix}}{\begin{bmatrix} -13 & -13 & 15 \\ 8 & 7 & -24 \\ -1 & 1 & 0 \end{bmatrix}} = \frac{2610}{1402} = 1.862A$$

$$I_b = \frac{D_2}{D}$$

$$I_c = \frac{D_3}{D}$$

أكمل الحل

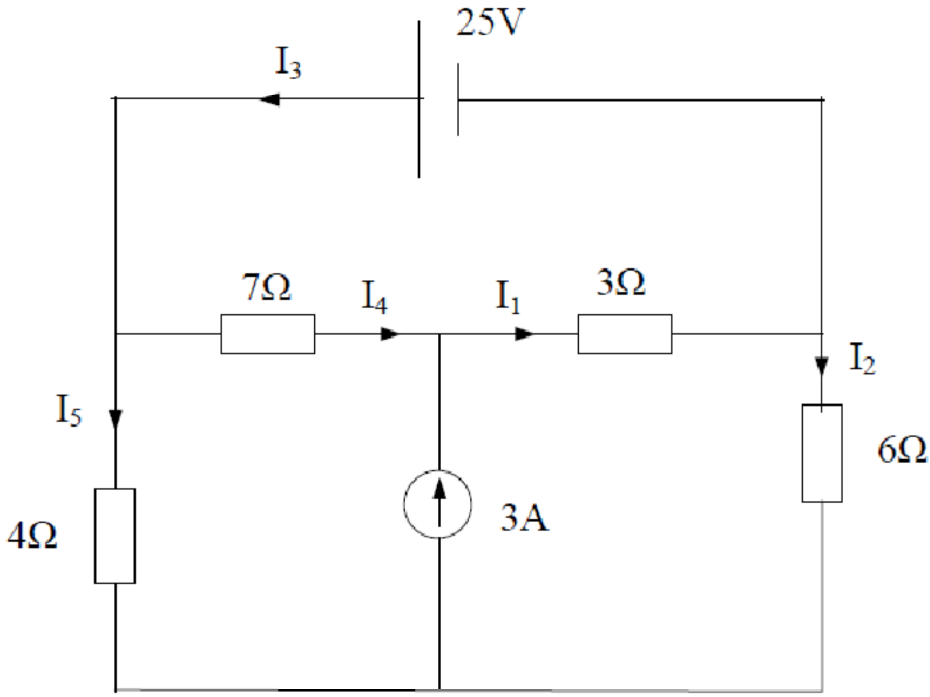
Lecture 14

Superposition Theorem

In any circuit network contain more than one sources (voltage or current) to find the current (or voltage) in a certain part of a network , remove the sources of the network and find the current (or voltage) in the existence of only one source each time. The resultant current (or voltage) will be the algebraic sum of current (or voltage) due to all sources when acting independently once a time .

(Removing the sources means:- Short circuiting the voltage source and open circuiting the current source**)** .

Example 1:- In the following circuit diagram, find all branch current's using superposition theorem:-



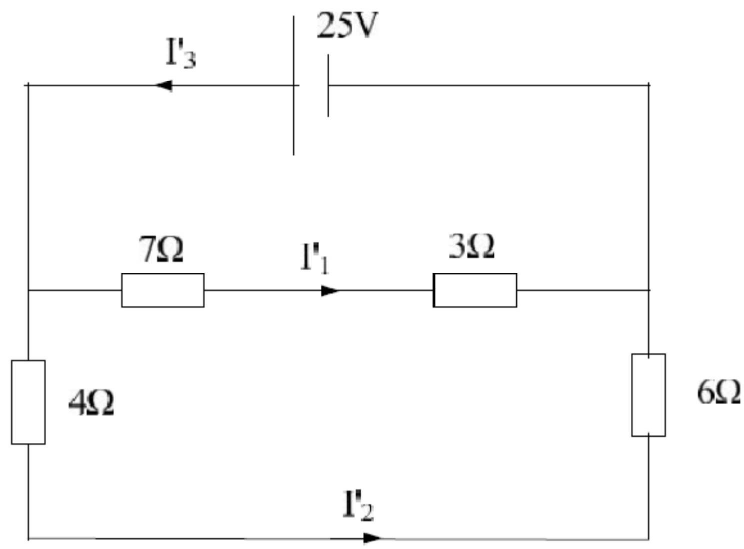
Solution :-

1.) Effect of 25 V source :-

$$I_1' = \frac{25}{7+3} = 2.5A$$

$$I_2' = \frac{25}{4+6} = 2.5A$$

$$I_3' = I_1' + I_2' = 5A$$



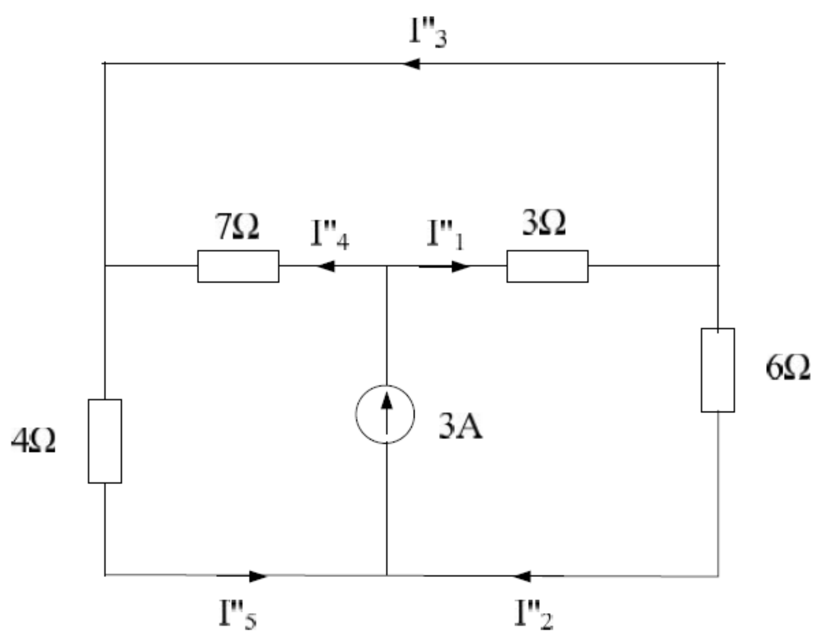
2.) Effect of 3 A source :-

$$I_1'' = 3 * \frac{7}{7+3} = 2.1A$$

$$I_4'' = 3 * \frac{3}{7+3} = 0.9A$$

$$I_2'' = 3 * \frac{4}{4+6} = 1.2A$$

$$I_5'' = 3 * \frac{6}{4+6} = 1.8A$$



$$I_3'' = I_1'' - I_2'' = I_5'' - I_4'' = 2.1 - 1.2 = 0.9A$$

3.) Superpose :-

$$I_1 = I_1' + I_1'' = 2.5 + 2.1 = 4.6A$$

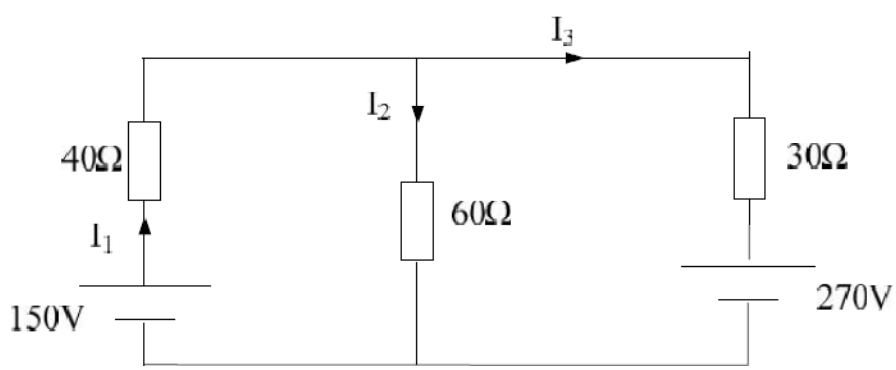
$$I_2 = I_2'' - I_2' = 1.2 - 2.5 = -1.3A$$

$$I_5 = I_2' + I_5'' = 2.5 + 1.8 = 4.3A$$

$$I_4 = I_1' - I_4'' = 2.5 - 0.9 = 1.6A$$

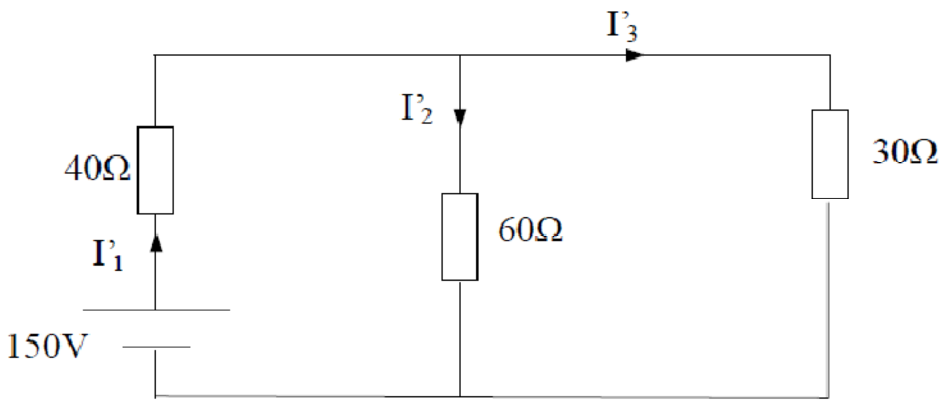
$$I_3 = I_3' + I_3'' = 5 + 0.9 = 5.9A$$

Example 2:- For the following circuit network, find the current in all branches, using superposition theorem:-



Solution:-

1.) Effect of 150 V source:-



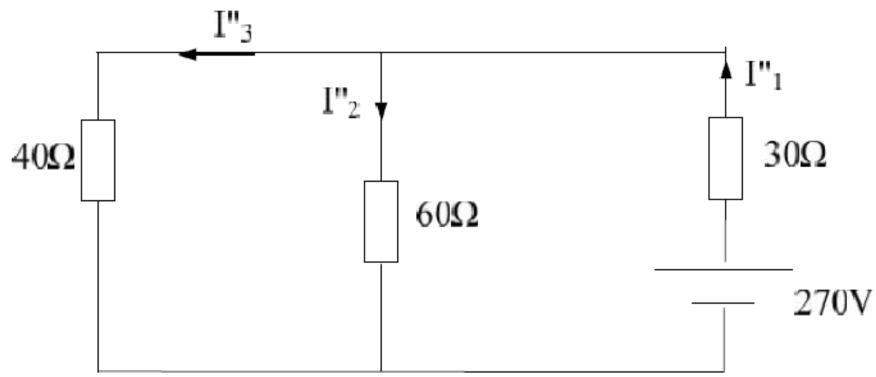
$$R_T = 40 + \frac{60*30}{60+30} = 60\Omega$$

$$I_1' = \frac{150}{60} = 2.5A$$

$$I_2' = 2.5*\frac{30}{60+30} = 0.83A$$

$$I_3' = 2.5*\frac{60}{60+30} = 1.67A$$

2.) Effect of 270 V source:-



$$R_T = 30 + \frac{40 * 60}{40 + 60} = 54\Omega$$

$$I_1'' = \frac{270}{54} = 5A$$

$$I_2'' = 5 * \frac{40}{40 + 60} = 2A$$

$$I_3'' = 5 * \frac{60}{40 + 60} = 3A$$

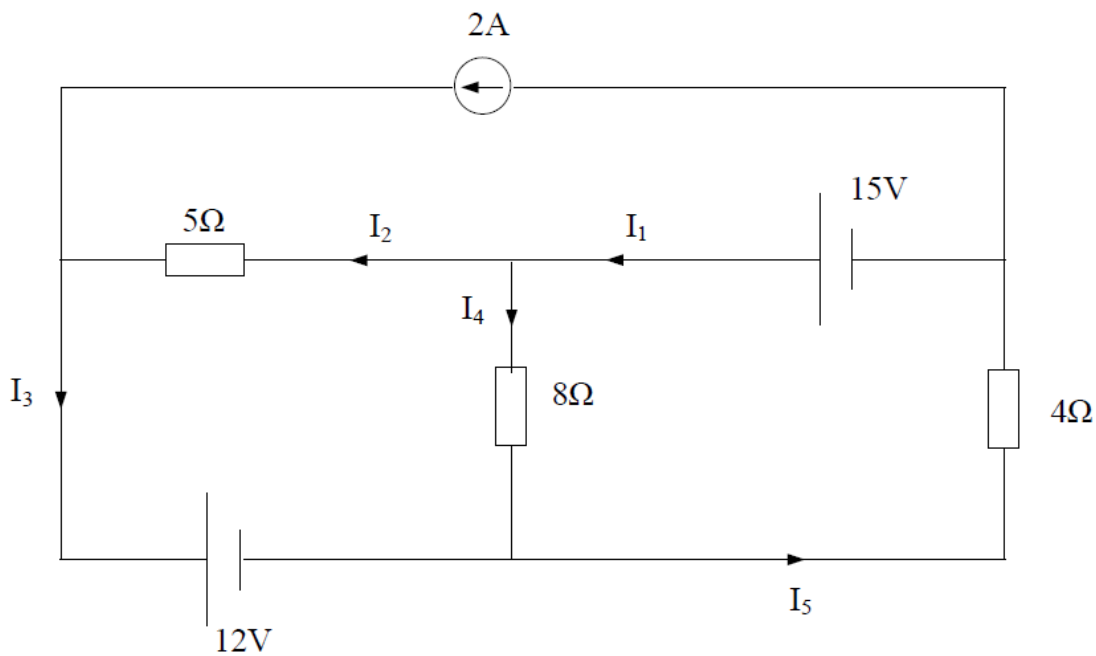
3.) Superpose :-

$$I_1 = I_1' - I_3'' = 2.5 - 3 = -0.5A$$

$$I_2 = I_2' + I_2'' = 0.83 + 2 = 2.83A$$

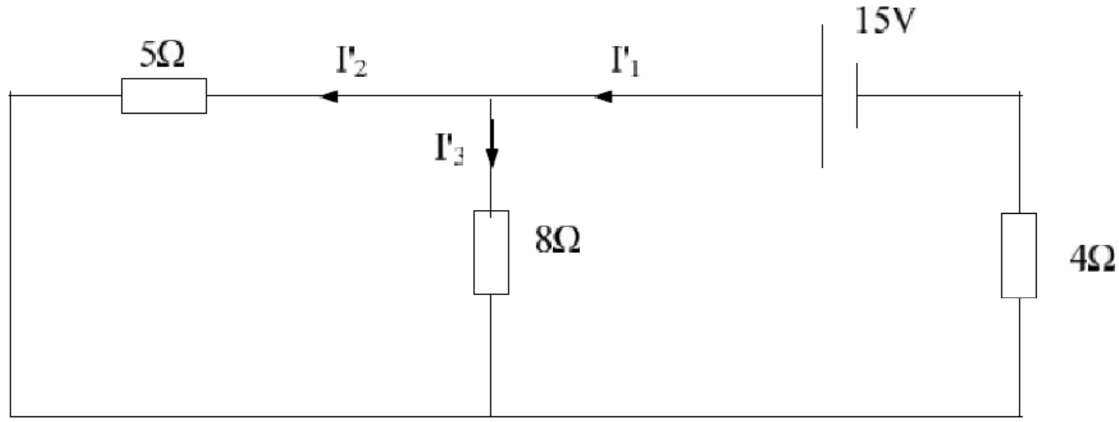
$$I_3 = I_3' - I_1'' = 1.67 - 5 = -3.33A$$

Example 3:- Find the current in all branch in the following circuit diagram:-



Solution:-

1.) Effect of 15 V source:-

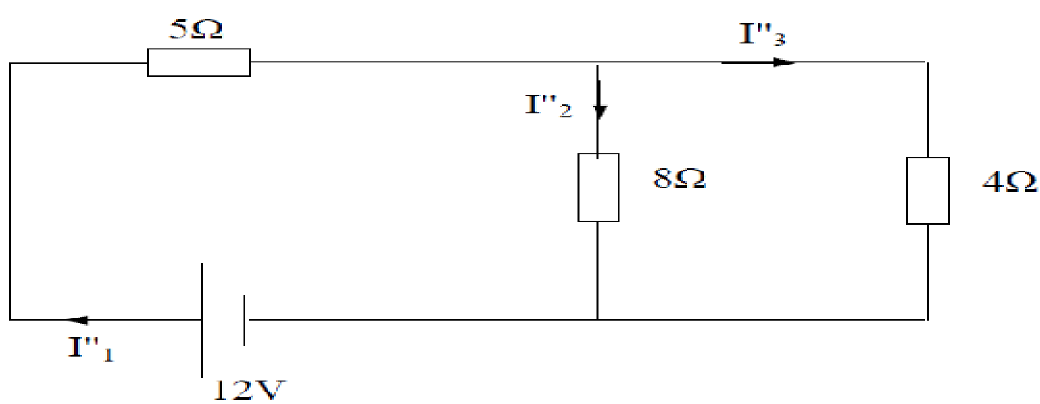


$$I_1' = \frac{15}{4 + \frac{5*8}{5+8}} = 2.12A$$

$$I_2' = 2.12 * \frac{8}{5+8} = 1.3A$$

$$I_3' = 2.12 * \frac{5}{5+8} = 0.82A$$

2.) Effect of 12 V source:-

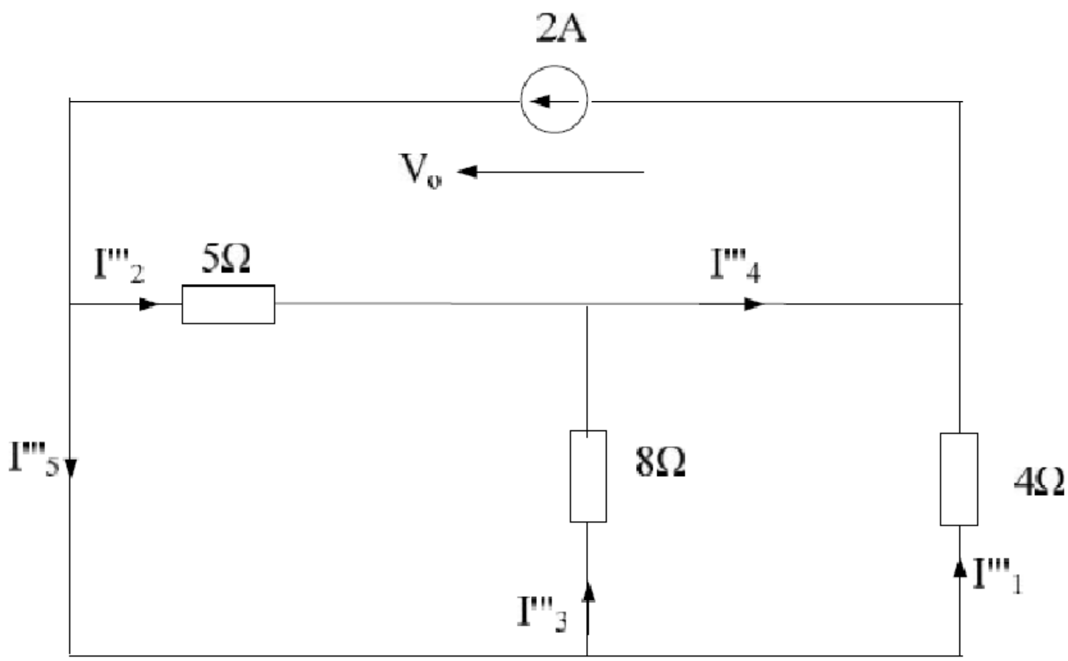


$$I_1'' = \frac{12}{5 + \frac{8 * 4}{8 + 4}} = 1.57A$$

$$I_2'' = 1.57 * \frac{4}{8 + 4} = 0.52A$$

$$I_3'' = 1.57 * \frac{8}{8 + 4} = 1.04A$$

3.) Effect of 2 A source:-



$$\frac{1}{R_T} = \frac{1}{8} + \frac{1}{4} + \frac{1}{5} \Rightarrow \therefore R_T = 1.74\Omega$$

$$V_o = 2 * R_T = 3.5V$$

$$I_1''' = \frac{V_o}{4} = 0.88A$$

$$I_2''' = \frac{V_o}{5} = 0.7A$$

$$I_3''' = \frac{V_o}{8} = 0.44A$$

$$I_4''' = 2 - I_1''' = 1.12A$$

$$I_5''' = 2 - I_2''' = 1.3A$$

3.) Superpose :-

$$I_1 = I_1' - I_3'' - I_4''' = 2.12 - 1.04 - 1.12 = -0.04A$$

$$I_2 = I_2' - I_1'' - I_2''' = 1.3 - 1.57 - 0.7 = -0.97A$$

$$I_3 = I_2' - I_1'' + I_5''' = 1.3 - 1.57 + 1.3 = 1.03A$$

$$I_4 = I_3' + I_2'' - I_3''' = 0.82 + 0.52 - 0.44 = 0.9A$$

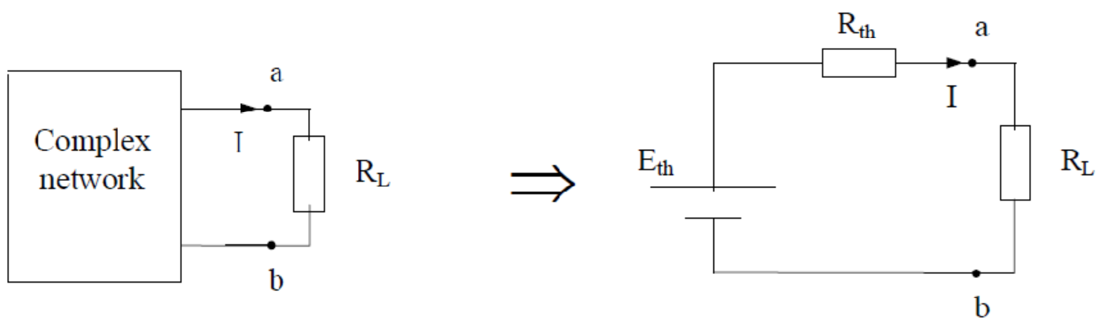
$$I_5 = I_1' - I_3'' + I_1''' = 2.12 - 1.04 + 0.88 = 1.96A$$

Lecture 15

Thevenin's Theorems:-

تستخدم في اغلب الاحيان اذا كان المطلوب ايجاد التيار او الفولتية في مقاومة محددة في الدائرة .

Any two terminal linear network can be replaced by an equivalent circuit of a voltage source (E_{th}) and a series resistor (R_{th}); as shown in figure below:-



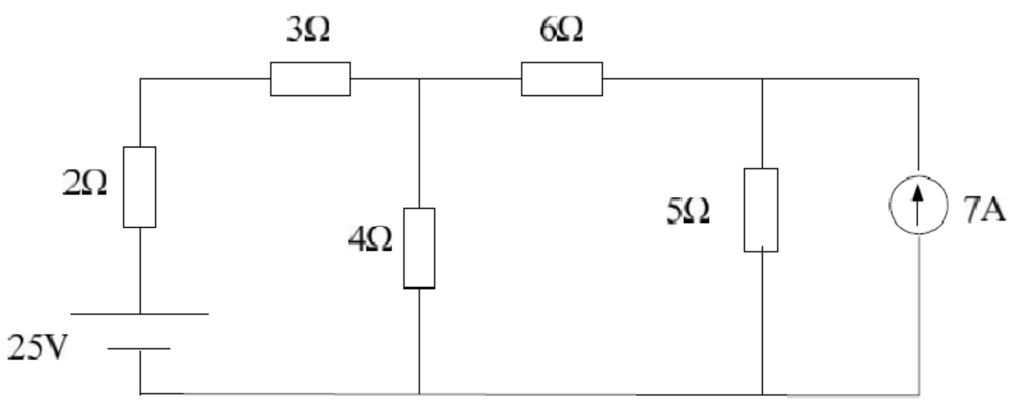
Hence; $I = \dfrac{E_{th}}{R_{th} + R_L}$

Steps to find E_{th} & R_{th} :-

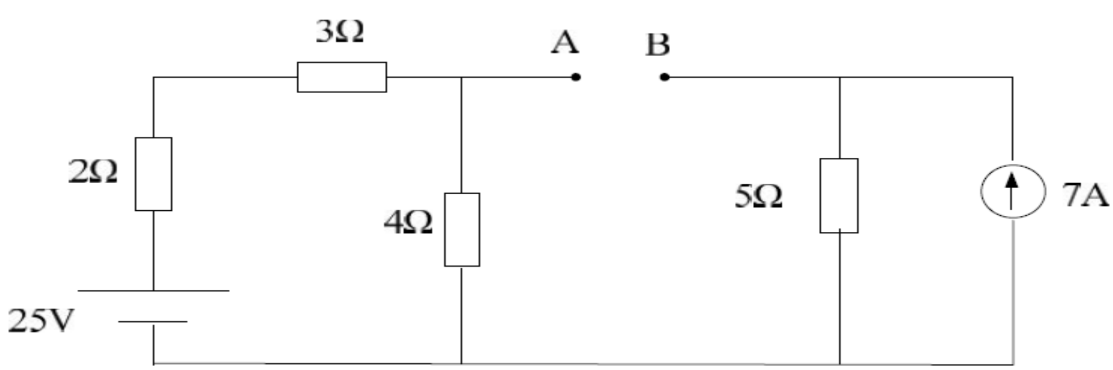
1. Remove that portion of the network across which the Thevenins equivalent circuit is to be find.
2. Mark the terminals of the remaining two – terminal network.
3. Calculate R_{th} by first setting all sources to zero (voltage sources are replaced by short circuits and current sources are replaced by open circuit), and finding the resultant resistance between the two marked terminals.

4. Calculate E_{th} by first returning all sources to their origin positions and finding the open circuit voltage between the marked terminals.
5. Draw the Thevenins equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.

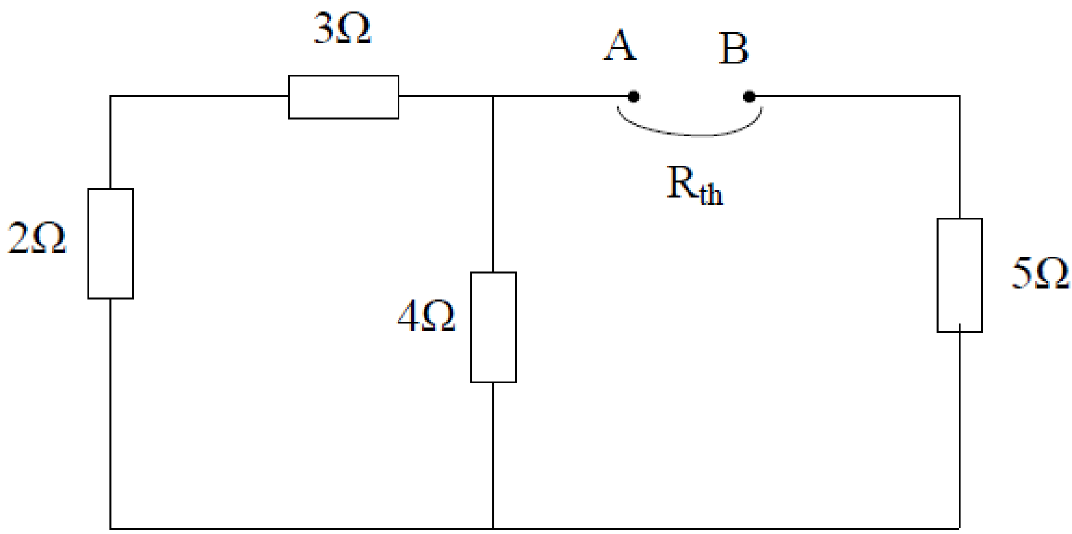
Example 1:- For the following circuit diagram, find the current in (6Ω) resistor?



Solution:-



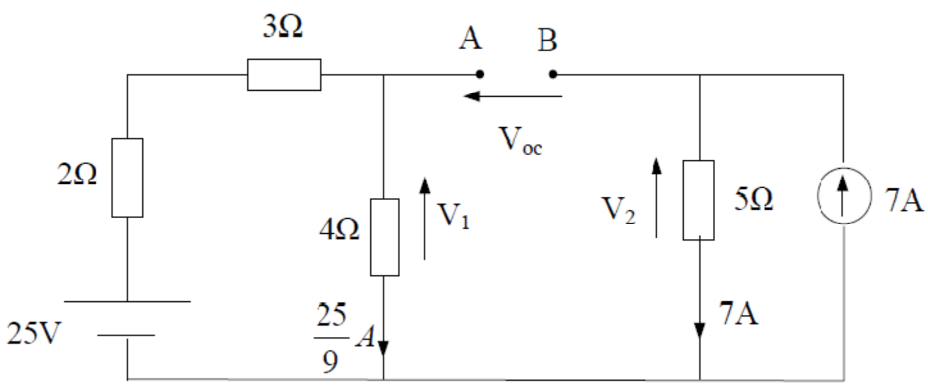
1.) Find R_{th} :



$$R_{th} = \{(2+3)//4\}+5$$

$$= \left\{\frac{5*4}{5+4}\right\}+5 = \frac{20}{9}+5 = 7.22\Omega$$

2.) Find E_{th} :

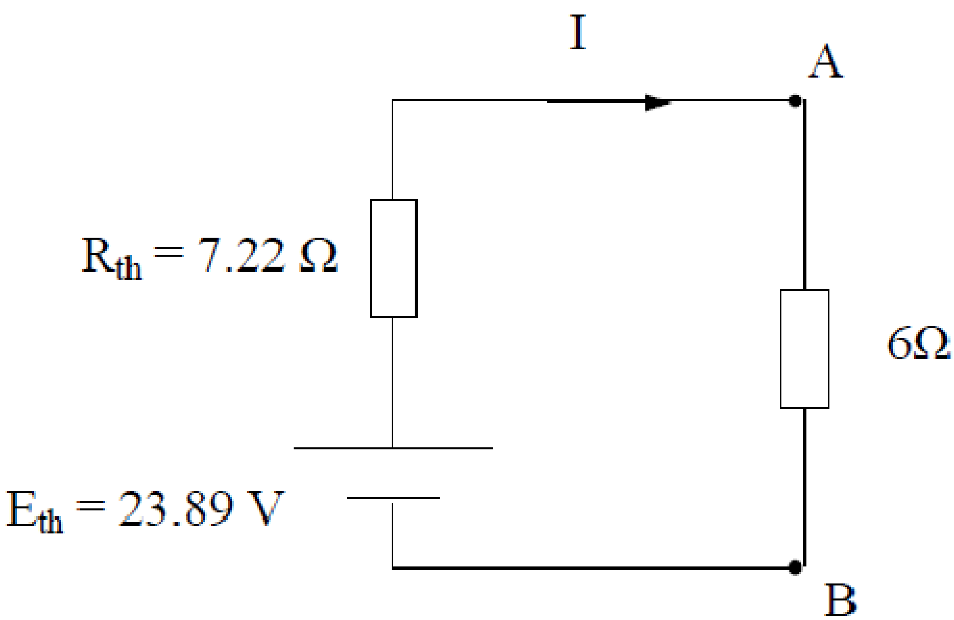


$$V_{oc} - V_1 + V_2 = 0$$

$$V_{oc} - \left(4 * \frac{25}{9}\right) + (7 * 5) = 0$$

$$V_{oc} = \frac{100}{9} - 35 = -23.89V$$

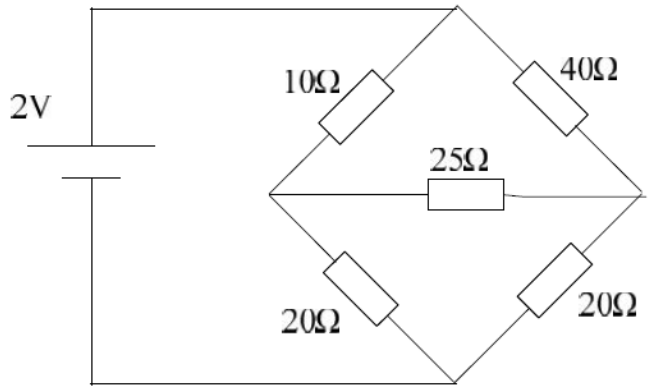
$$\therefore E_{th} = 23.89V$$



$$I = \frac{E_{th}}{R_{th} + R_L}$$

$$= \frac{23.89}{7.22 + 6} = 1.8A$$

Example 2:- Find the current in the 25Ω resistor for the following circuit network?

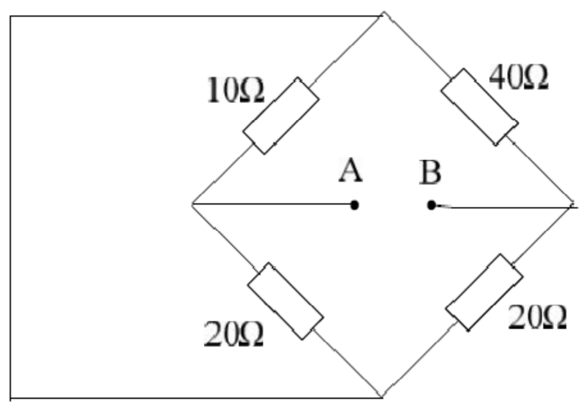


Solution:-

1.) Find R_{th} :

$$R_{th} = (10 // 20) + (40 // 20)$$

$$\therefore R_{th} = \frac{10*20}{10+20} + \frac{40*20}{40+20} = 20\Omega$$

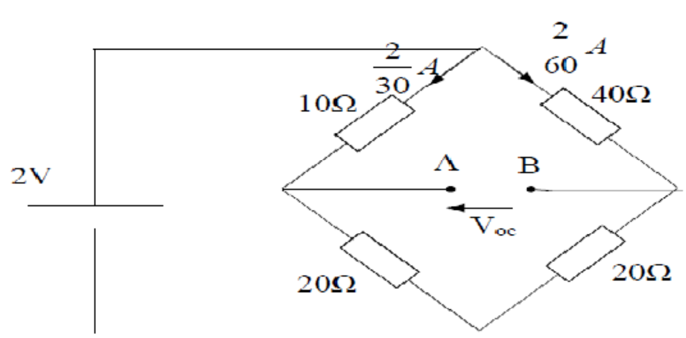


2.) **Find E_{th} :**

$$V_{oc} + \left(10*\frac{2}{30}\right) - \left(40*\frac{2}{60}\right) = 0$$

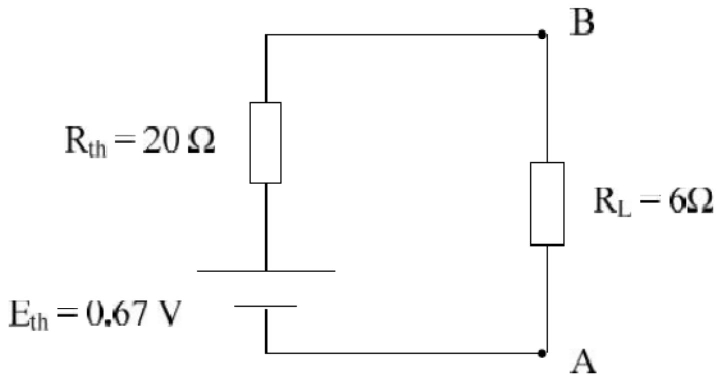
$$V_{oc} = \frac{80}{60} - \frac{20}{30} = \frac{40}{60} = 0.67V$$

$$\therefore E_{th} = 0.67V$$

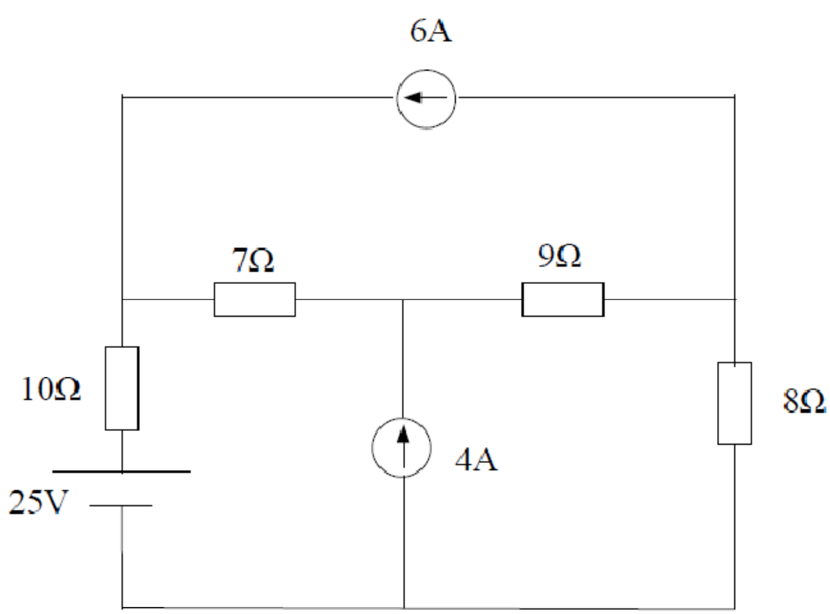


$$I = \frac{E_{th}}{R_{th} + R_L}$$

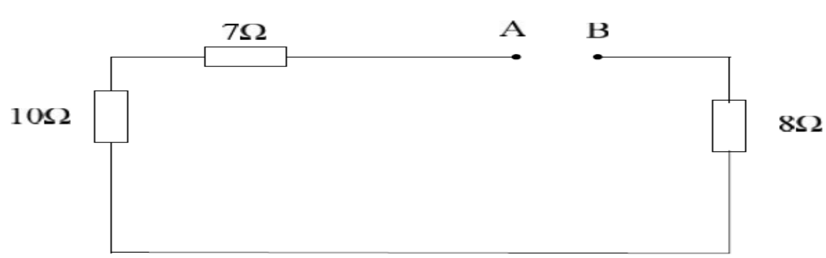
$$\therefore I = \frac{0.67}{20 + 25} = \frac{0.67}{45} A$$



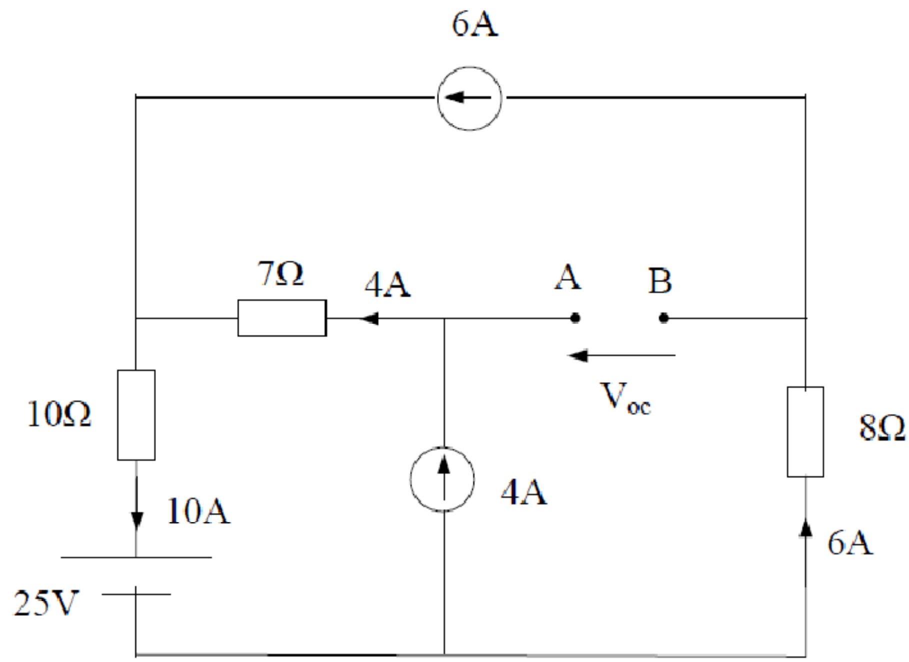
Example 3:- Find I in the (9Ω) resistor for the following cct. diagram?



Solution :-



$R_{th} = 7 + 10 + 8 = 25\Omega$

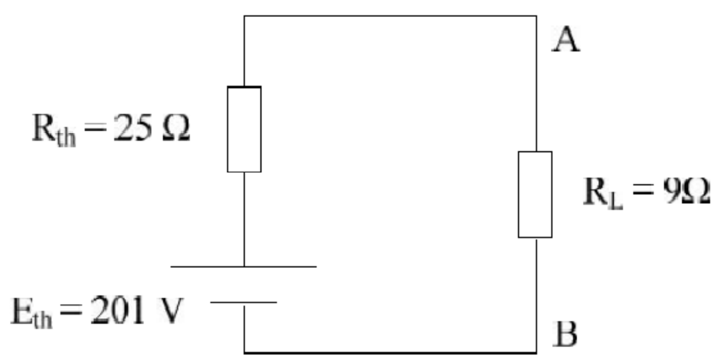


$$V_{oc} - (4*7) - (10*10) - 25 - (8*6) = 0$$

$$\therefore V_{oc} = 201V$$

$$I = \frac{E_{th}}{R_{th} + R_L}$$

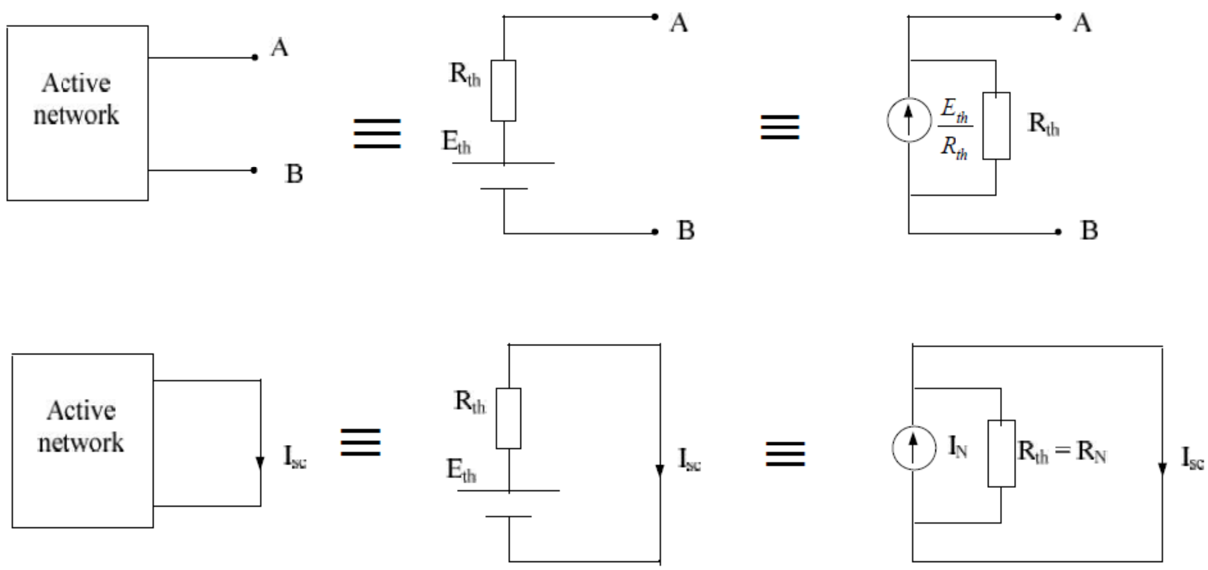
$$= \frac{201}{25+9} = 5.91A$$



Lecture 16

Norton's Theorems:-

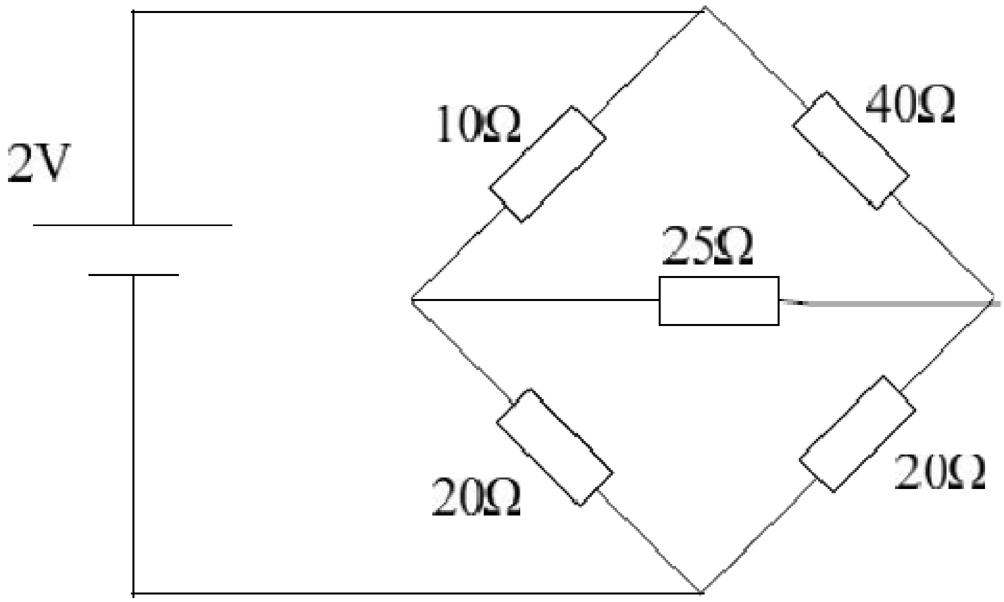
Any two terminal linear network can be replaced by an equivalent circuit consisting of a current source and a parallel resistor.



$R_N = R_{th}$ as before .

$I_N = I_{sc}$ = short circuit current between the two terminals of the active network.

Example 1:- Find the current in 25Ω resistor for the following circuit network using Norton's Theorem?

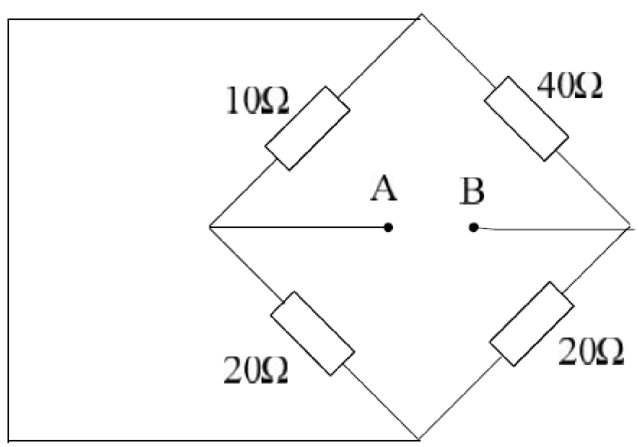


Solution:-

First find R_N :-

$$R_N = (10 // 20) + (40 // 20)$$

$$= \frac{10 * 20}{10 + 20} + \frac{40 * 20}{40 + 20} = 20\Omega$$

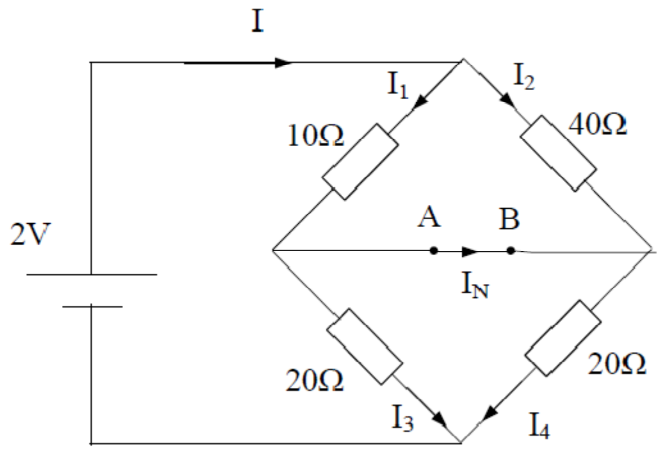


Second find I_N :-

$$I = \frac{2}{\frac{10 * 40}{10 + 40} + \frac{20 * 20}{20 + 20}} = \frac{2}{8 + 10} = \frac{1}{9} A$$

$$I_1 = \frac{1}{9} * \frac{40}{50} \;\&\; I_2 = \frac{1}{9} * \frac{10}{50}$$

$$I_3 = \frac{1}{9} * \frac{20}{40} \;\&\; I_4 = \frac{1}{9} * \frac{20}{40}$$

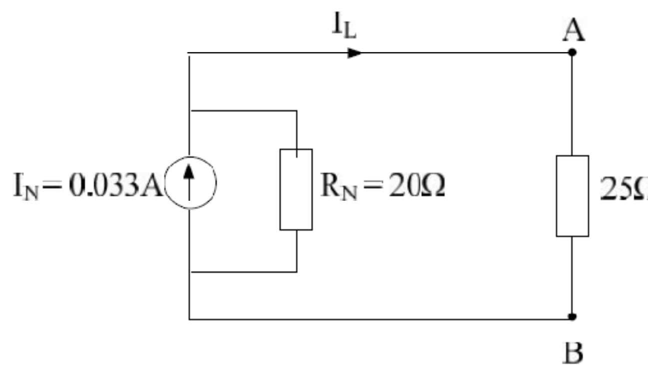


KCL at A

$$I_1 - I_N - I_3 = 0$$

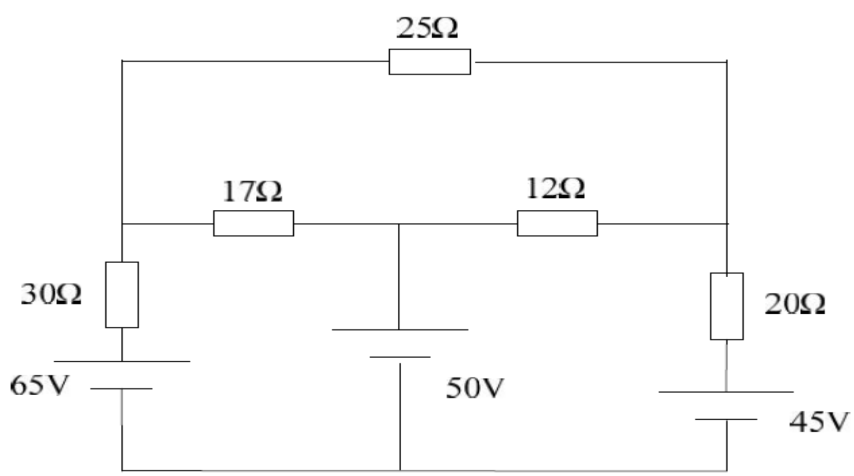
$$\therefore \quad I_N = I_1 \quad I_3$$

$$= \left(\frac{1}{9} * \frac{40}{50}\right) - \left(\frac{1}{9} * \frac{20}{40}\right) = 0.033A$$



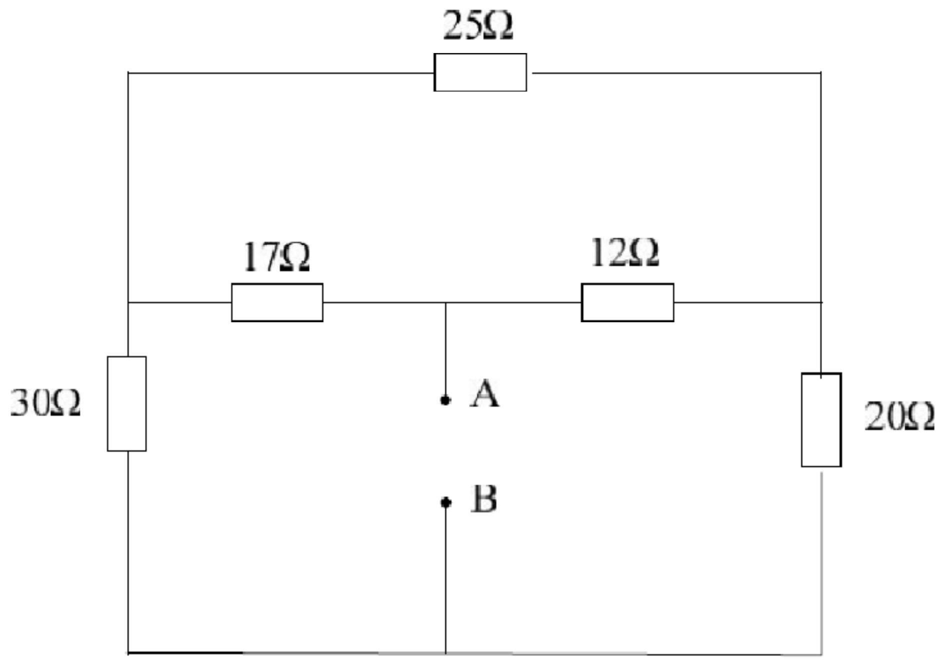
$$\therefore I_L = 0.033 * \frac{20}{20 + 25} = 0.0147A$$

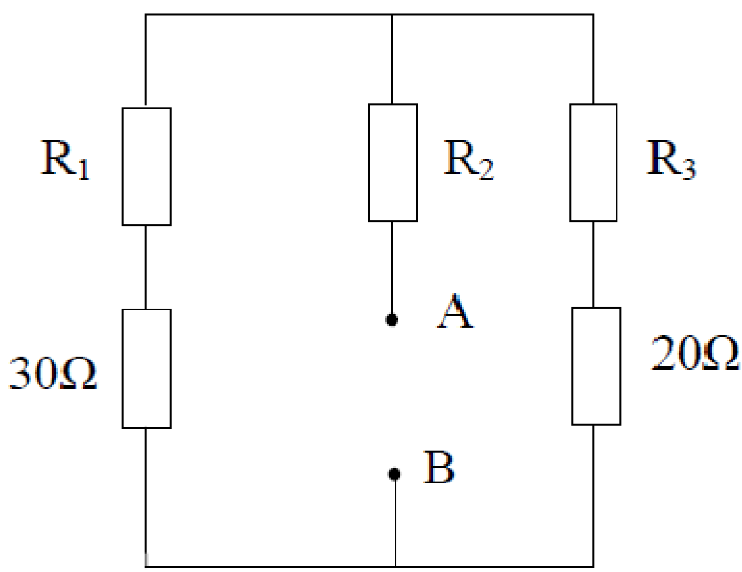
Example 2:- Find I in 50v voltage source, for the following circuit using Norton's Theorem?



Solution:-

1.) Find R_N :-



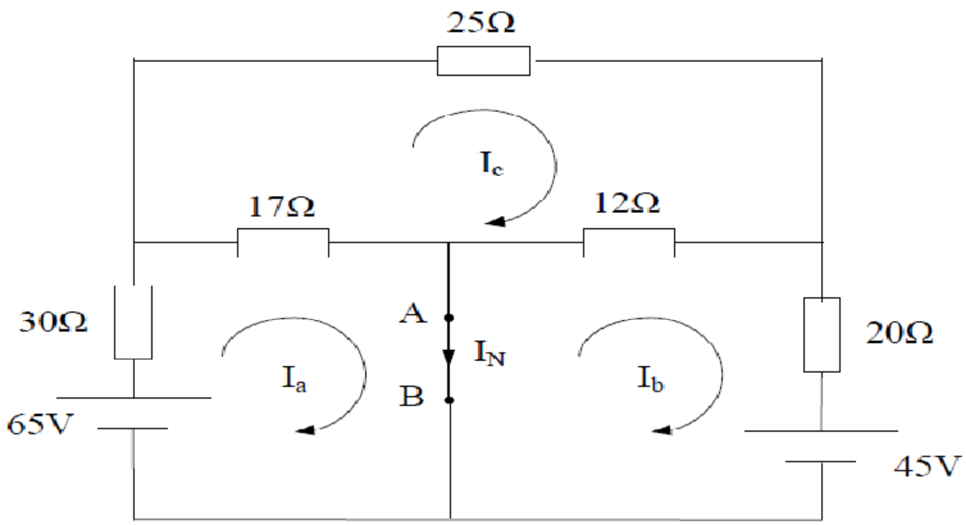


$$R_1 = \frac{17*25}{54} = 7.8\Omega \quad , \quad R_2 = \frac{12*17}{54} = 3.78\Omega \quad , \quad R_3 = \frac{25*12}{54} = 5.56\Omega$$

$$R_N = [(R_1 + 30)//(R_3 + 20)] + R_2$$

$$= [37.8//25.56] + 3.78 = 19\Omega$$

2.) Find I_N :-



$$-47I_a + 17I_c + 65 = 0$$

$$-32I_b + 12I_c - 45 = 0$$

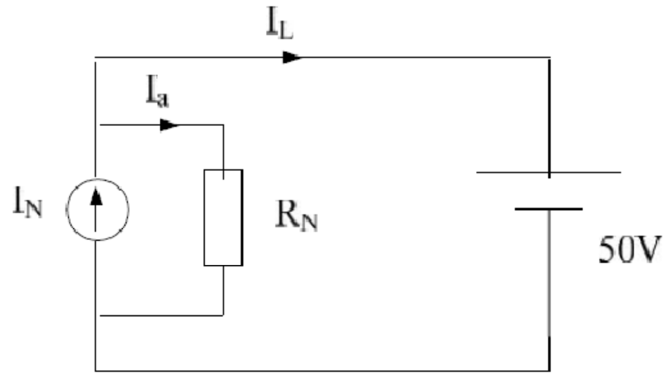
$$-54I_c + 17I_a + 12I_b = 0$$

After find I_a , I_b , I_c

$$I_N = I_a - I_b$$

$$I_N - I_a - I_L = 0$$

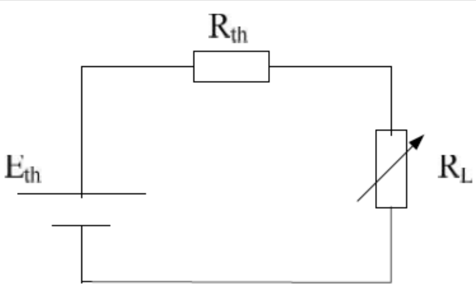
$$I_L = I_N - I_a = I_N - \frac{50}{R_N} = I_N - \frac{50}{19}$$

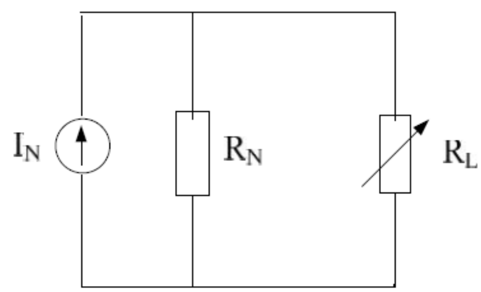


Lecture 17

Maximum Power Transfer:-

A load will receive maximum power from a d.c. network when its total resistive value is exactly equal to the Thevenin resistance of the network.

<table>
<tr><th>For Thevenin cct.</th><th>Nortan cct.</th></tr>
<tr><td>


</td><td>


</td></tr>
<tr><td>

For Max. power

$$\boxed{R_L = R_{th}}$$

$$P_{L_{max.}} = I_L^2 R_L = \left(\frac{E_{th}}{R_{th}+R_L}\right)^2 * R_L$$

$$= \frac{E_{th}^2}{4R_{th}^2} * R_{th}$$

$$\boxed{\therefore P_{L_{max.}} = \frac{E_{th}^2}{4R_{th}}}$$

</td><td>

$$\boxed{R_L = R_N}$$

$$P_{L_{max.}} = I_L^2 R_L = \left(I_N \frac{R_N}{R_N+R_L}\right)^2 * R_L$$

$$= I_N^2 \frac{R_N^2}{4R_N^2} * R_N$$

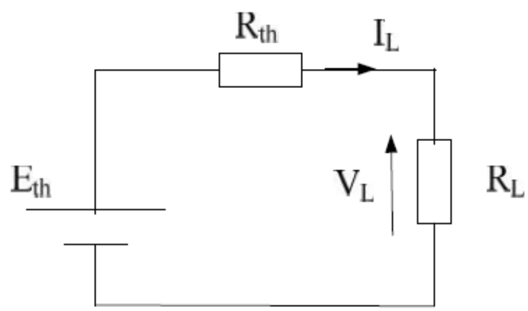
$$\boxed{\therefore P_{L_{max.}} = \frac{I_N^2 R_N}{4}}$$

</td></tr>
</table>

Under Max. Power transfer conditions, the efficiency is:-

$$\eta\% = \frac{P_o}{P_i} * 100\%$$

$$= \frac{V_L I_L}{E_{th} I_L} * 100\% = \frac{V_L}{E_{th}} * 100\%$$



$E_{th} = V_L + R_{th}\, I_{th}$

$\because\ R_{th} = R_L$ (for max. power transfer)

$E_{th} = V_L + R_L\, I_L$

$= V_L + V_L = 2V_L$

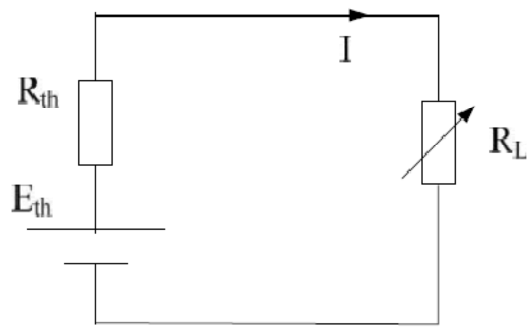
$$\eta = \frac{V_L}{E_{th}} * 100\% = \frac{V_L}{2V_L} * 100\% = 50\%$$

The efficiency will always be 50% under max. power transfer conditions .

<u>* Practical example:-</u>

$$P_L = I^2 R_L$$

$$= \left(\frac{E}{R_{th} + R_L} \right)^2 * R_L$$



Let $R_{th} = 3\Omega$ & $R_L = 1\Omega$ & $E_{th} = 15$ V

$$\therefore P_L = \left(\frac{15}{3+1} \right)^2 * 1 \approx 14W$$

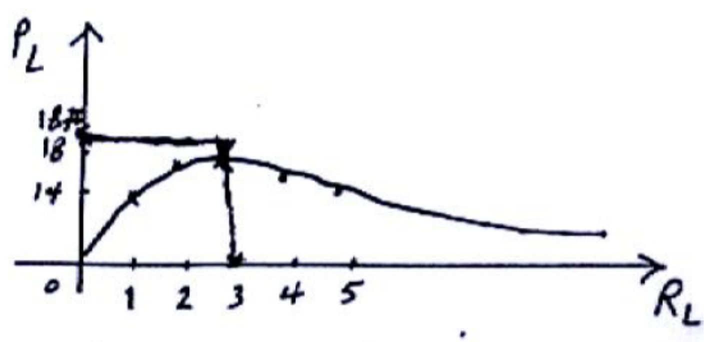
For $R_L = 2\Omega$ $\Rightarrow P_L = \left(\frac{15}{5} \right)^2 * 2 = 18W$

For $R_L = 3\Omega$ $\Rightarrow P_L = \left(\frac{15}{6} \right)^2 * 3 = 18.75W$

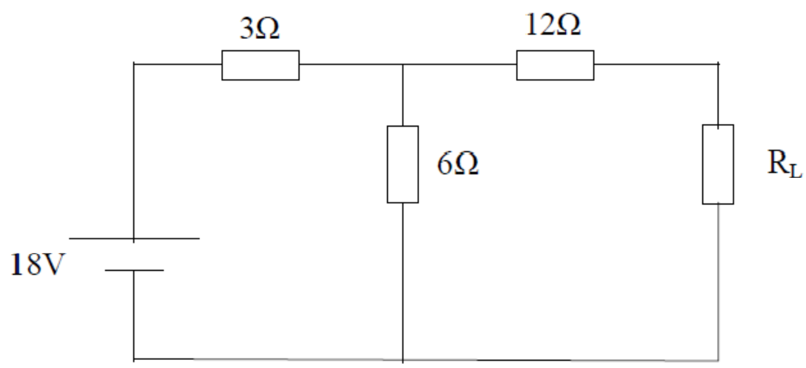
Note that when $R_L = R_{th}$, we get the max. power of P_L .

Hence $P_L = \frac{1}{2} P_{in} = \frac{1}{2} EI$

or $P_{in} = 2\, P_L$

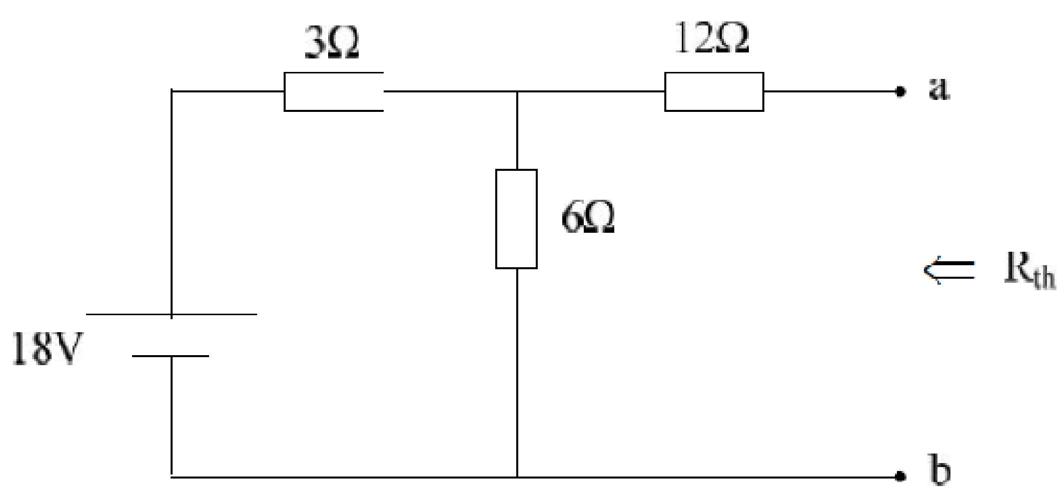


Example 1:- Find the value of R_L for maximum power transfer to R_L , and determine the power delivered under these conditions ?



Solution:-

First remove R_L , and find the equivalent resistance (R_{th})



$$R_{th} = (3 // 6) + 12$$

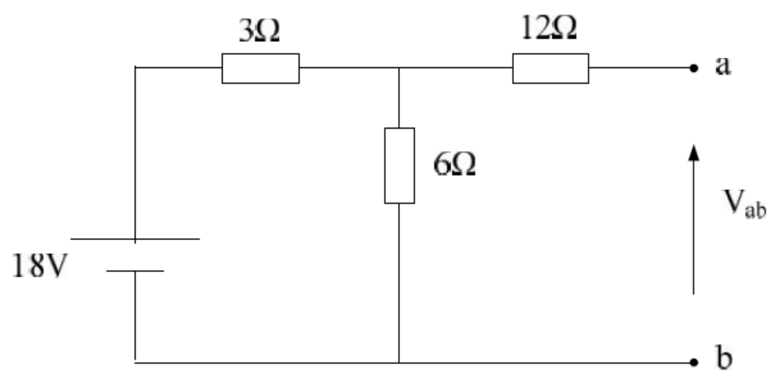
$$\therefore R_{th} = \frac{3 * 6}{3 + 6} + 12 = 14\Omega$$

For Max. power $R_L = R_{th}$

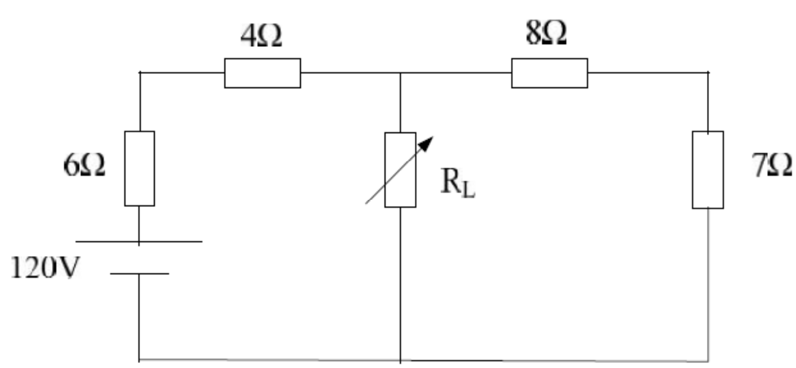
$$\therefore R_L = 14\Omega$$

$$E_{th} = V_{ab} = \frac{18 * 6}{6 + 3} = 12V$$

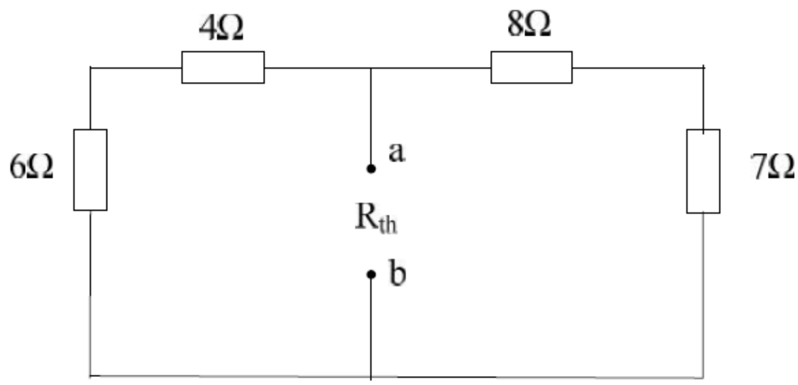
$$\therefore P_{max.} = \frac{E_{th}^2}{4R_{th}} = \frac{(12)^2}{4 * 14} = 2.57W$$



Example 2:- Find the value of R_L for the following cct. for max. power transfer, and find P_L?



Solution:-

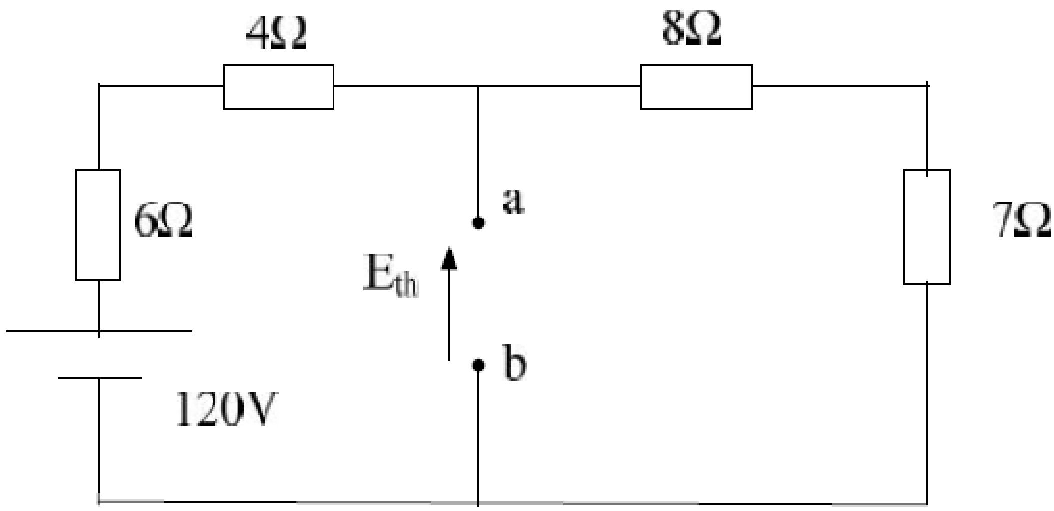


$$R_{eq.} = R_{th} = (8+7)//(6+4)$$

$$= \frac{15*10}{15+10} = 6\Omega = R_L$$

$$V_{oc} = V_{ab} = E_{th} = \frac{120}{(6+4+8+7)}*(8+7)$$

$$= \frac{120}{25}*15 = 72V$$



$$P_L = I^2 R_L = \left[\frac{72}{(6+6)}\right]^2 * 6 = 216W$$

or

$$P_L = \frac{E_{th}^2}{4R_{th}} = \frac{7^2}{4*6} = 216W$$